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# Performance Assessment of a Model-based DC Motor Scheme

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**Abstract:** The separately-excited DC motor is a high-performance variable speed drive with industrial applications such as in robotics, actuation, control and guided manipulation due to its precision, simplicity, continuous control feature and wide speed range. There is therefore, the need to regulate and drive the motor at desired speed in the presence of uncertainties and motor parameter variations. However, there is not a single optimal controller for the machine. Hence, one major control objective is to find the perfect trade-off between performance and robustness. The conventional Proportional-Integral-Derivative (PID) controller is not the optimal control strategy to achieve this objective because of its oscillatory response, inability to cope with changing operating conditions and sensitivity to motor parameter variations. Hence, this paper proposes a performance review of the Internal Model Control (IMC) feedback scheme in order to find the optimal controller setting that will ensure tight control which is high performance subject to acceptable robustness. This optimal controller setting will ensure excellent reference speed tracking, fast and non-oscillatory response subject to acceptable robustness to motor parameter variations. Computer simulations are also presented to show the practicability of the study.

Keywords: DC motor control; Feedback control; Internal model control; IMC-tuned PID; Robustness.

## 1. INTRODUCTION

The DC motor is a type of electrical machine that converts electrical energy into mechanical form. It achieves this using direct current in both stator and rotor. It has widespread industrial applications such as in machine tools, robotic manipulators, household appliances, servomotors, paper mills, and propulsion systems for electric vehicles. This is due to its precision, simplicity, continuous control features and wide speed range. As a distinguishing feature of DC machines, the field circuit and the armature circuit can be interconnected in several ways to deliver a wide range of performance characteristics. A variant of DC motors is the separately-excited motor where the field circuit and the armature circuit are fed with independent voltage sources. This arrangement helps to achieve variable speed control and full torque at all speeds, where the field can be connected to a constant voltage source to ensure full torque is available at all speeds while the armature can be connected to a variable voltage DC source to achieve speed control. This type of high-performance variable speed drives is vital for industrial applications such as robotics, actuation, control and guided manipulation where precision movements are essential [1-5].

A major performance feature of the DC motor is the torque-speed characteristic, which is a description of how the torque produced by the machine varies as a function of the speed of rotation of the motor for steady speeds. Hence, a servo drive system is rated high performance when the shaft speed can be made to follow a desired reference at all times. Nevertheless, there is not a single optimal controller for the machine. Hence, one major control objective is to find the perfect trade-off between performance and robustness. For any given robustness level, there exists an optimal controller setting that maximizes the performance which is defined as accurate reference tracking, a fast and non-oscillatory response to a set point change [6].

Another control objective for the machine is to regulate and drive the motor at desired speed in the presence of parameter variations and changing operating conditions [1,5]. It is thus desired that the speed control system for the DC motor should satisfy the following performance criteria: excellent set point tracking subject to acceptable robustness i.e. insensitivity to motor variations [7]. The PID (Proportional, Integral and Derivative) controller, a conventional and popular feedback control scheme, used in the industry for control gives a performance that is unsatisfactory because of its oscillatory response and sensitivity to motor parameter variations. It is also limited in its ability to provide predictive control action to compensate for changing operating conditions. The PID controller is thus, not the optimal control strategy to ensure tight control (good performance subject to satisfactory robustness) for variable speed drive applications [2,6-8]. This presents the need to design a robust and high-performance model-based controller that will achieve this desired objective for the DC motor speed.

Hence, the main contribution of this paper is to determine the optimal model-based controller settings that will ensure high performance subject to acceptable robustness. The Model-based PID controller using the derived model of the system will effectively capture and address control challenges presented by the hidden dynamics, complexities and nonlinearities of the DC motor and will provide a better control scheme/regime compared to the conventional. However, because the controller is based on an approximate model of the motor there could be limitations of model inaccuracy (plant-model mismatch) and external disturbances for the model-based controller.

Therefore, a model will be developed, then based on desirable control objectives for the speed of the DC motor optimal settings for the PID control algorithm is obtained. The proposed control scheme should provide for tight control. This paper, therefore, has the following objectives:

- To design and build an optimal IMC-tuned PID (feedback) controller to guarantee high performance subject to very acceptable robustness of the DC motor speed.
- To compare and evaluate the performance of the model-based controller for various values of the desired closed-loop time constant $\tau_c$. This is the tuning parameter to be optimally designed for the proposed control scheme.
- To better understand, using the measures of accuracy and speed of response as well as stability margin, how the choice of the tuning parameter affects performance and robustness and hence make a recommendation of the optimal controller settings for the DC motor.

Speed control of a separately-excited DC motor can be achieved via the following methods: armature voltage control, field flux control and armature resistance control. In this paper armature voltage speed control technique, where the armature terminal voltage is varied to change the speed, will be applied on a single separately-excited DC motor (SEDCM).

## 2. BACKGROUND

Recently considerable effort has been devoted to DC motor speed control using different approaches, like PID, fractional-order dynamics, Sliding Mode Control (SMC), but no attempt has been made to use a model of the DC motor to obtain optimal tuning settings for the PID controller.

An approach based on linear parametric models, derived using state estimation and system identification techniques is used to control the motor, but their displacement response had oscillations and due to plant-model mismatch the controller performance is reduced [9]. An approach incorporated fractional-order dynamics to improve the performance and change the dynamics of the entire DC motor by eliminating the original influence of the conventional PID, without making internal changes into the system. But indices like settling time, rise time, steady-state error and overshoot were not use to evaluate its performance [10]. An effective and profound approach based on SMC and Adaptive PID with SMC gives a good motor speed performance with acceptable robustness. The only flaw with this technique is that the performance characteristics however are not as optimized as a larger rise time is observed with the SMC method than with the conventional, and there is the chattering phenomenon where the control signal switches between two structures leading to high frequency oscillations [2,11].

An approach using Universal Learning Networks (ULNs), a class of Neural Networks, for the identification and control of a separately excited DC motor, loaded with a centrifugal pump, gave an overall satisfactory performance. Firstly, offline a Universal Learning Network Identifier (ULNI) is trained to emulate the dynamic behaviour of the DC motor system, using the forward propagation algorithm. This identifier is then used online to train a Universal Learning Network Controller (ULNC) that regulates the motor speed to track a desired reference signal. Performance indicators, like settling time, rise time, steady-state error and overshoot were not discussed here [12]. The review of literature reveals that even though there are many control strategies for the control of the speed of a DC motor, there is still a gap with respect to the need to improve the performance and robustness of a model-based control scheme for the motor. The above stated limitations with literature will be handled by this paper.

This paper is organized as follows. First a mathematical model of the separately-excited DC motor is derived in Section 3. The proposed IMC-based PID control technique designed using a model of the motor is introduced in Section 4. Section 5 presents the simulation results for the proposed controller, for various choice of the tuning parameter as well as an evaluation of the performance. Finally, and in Section 6, major conclusions are drawn.

## 3. MATHEMATICAL MODEL OF THE SEPARATELY-EXCITED DC METHOD

The separately excited DC motor is a machine wherein the field and armature circuit have independent voltage sources. The equivalent circuit showing the electromechanical arrangement of this type of DC motor is shown in Figure 1. The motor speed consistent with the rated armature voltage is its rated speed. Armature voltage control serves to smoothly vary the motor speed from zero to its rated speed (base speed) by varying the armature voltage in the constant torque region. The armature current increases with increase in the armature voltage, which consequently leads to an increase in the development torque and, hence the motor speed [1,4,13]. The dynamics of the motor and its load are represented by the following set of differential equations:

$$v_a(t) = R_a i_a(\mathrm{t}) + L_a \frac{di_a(t)}{dt} + K\omega(t) \tag{1}$$

$$T_e = Ki_a = B\omega(t) + J\frac{d\omega(t)}{dt} + T_l(t) \tag{2}$$

where the $T_e = Ki_a$, is the development electrical torque, $E_b = K\omega$, is the back EMF, the constant $K$ denotes the torque/back EMF constant, $B$ denotes the viscous friction coefficient/motor damping, $J$ is the motor shaft inertia, $\omega$ is the motor speed and $T_L$ is the load torque. Typically, $T_L$ is either constant or some function of the speed ω in a motor [1].

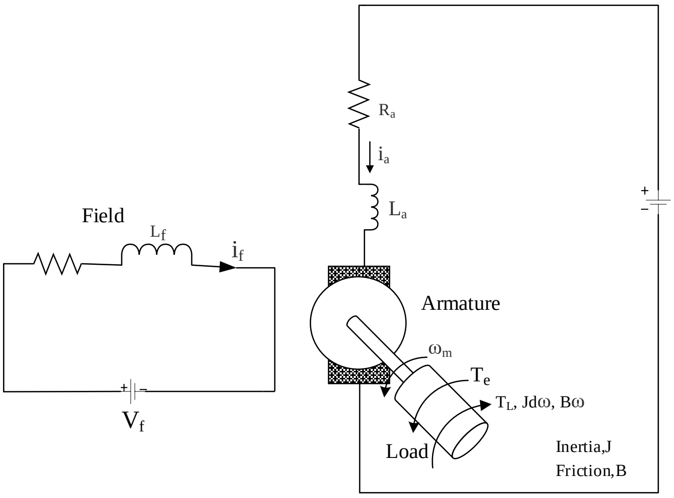


Figure 1. Equivalent circuit of the separately excited DC motor

To derive the input-output relationship between the armature voltage $v_a$ and the motor speed $\omega$, the Laplace transformation of Equations (1) and (2) yields:

$$V_a(s) = R_a I_a(s) + sL_a I_a(s) + K\omega(s) \tag{3}$$

$$KI_a(s) = B\omega(s) + sJ\omega(s) + T_L(s) \tag{4}$$

If current in the armature $I_a$ is made the subject from Equation (4) and resubstituted in Equation (3) we have the system transfer function that depicts the input-output relationship between the motor speed $\omega$, armature voltage $V_a$ and load torque $T_L$:

$$\omega(s) = \frac{\frac{K}{L_aJ}}{s^2 + \left(\frac{R_aJ + L_aB}{L_aJ}\right)s + \left(\frac{R_aB + K^2}{L_aJ}\right)} V_a - \frac{sL_a + R_a}{s^2 + \left(\frac{R_aJ + L_aB}{L_aJ}\right)s + \left(\frac{R_aB + K^2}{L_aJ}\right)} T_L \tag{5}$$

$$\omega(s) = G_m V_a + G_d T_L \tag{6}$$

where $G_m$ is the transfer function that depicts the relationship between the motor speed $\omega$ and the armature voltage $V_a$, while $G_d$ depicts the relationship between the motor speed $\omega$ and load torque, the disturbance.

For this paper it is assumed that the load torque applied to the DC motor is negligible, hence $G_d = 0$. Hence Equation (5) is rewritten to be:

$$\omega(s) = \frac{\frac{K}{L_aJ}}{s^2 + \left(\frac{R_aJ + L_aB}{L_aJ}\right)s + \left(\frac{R_aB + K^2}{L_aJ}\right)} V_a \tag{7}$$

The expression in Equation (7) can be represented in the block diagram shown in Figure 2.

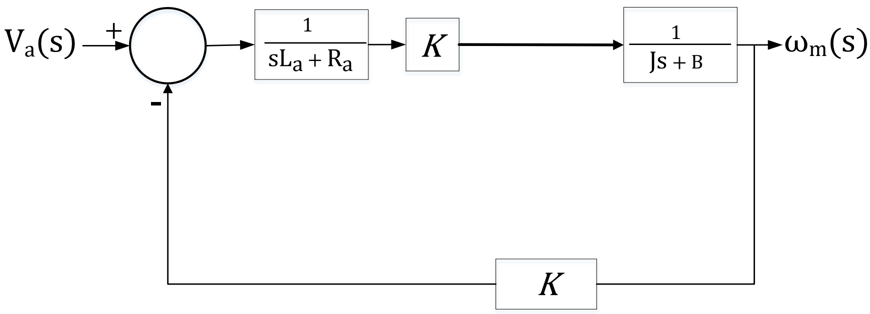


Figure 2. Block diagram configuration of a SEDCM

## 4. PROPOSED APPROACH

In the following subsections, an IMC-based feedback controller designed using a model of the motor is developed. An evaluation of the performance criteria for the controller is also described.

### 4.1 IMC-tuned Feedback Control

The IMC control scheme is a model-based control strategy [14], that is based on an assumed system model which gives rise to analytical expressions for the controller settings. The benefit of this approach is that it permits model uncertainty and trade-offs between performance and robustness to be considered in a more systematic manner. Instead of setting a control structure and then trying to extract optimality from this controller, as is the case with conventional PID control, the approach of this work is to postulate a model and state desirable control objectives for the speed of the DC motor. Then, from these the suitable control structure and parameters will be obtained in a straightforward manner [7,14].

The simplified block diagram of the IMC approach is shown in Figure 3 while that of IMC-derived standard feedback control is shown in Figure 4. Here $G_c^*$ is the Internal Model Control algorithm while $G_c$ is the equivalent feedback (PID) control algorithm derived using the IMC tuning rules. In Figure 3, the assumed model of the DC motor $\tilde{G}_m$ and the controller output are used to calculate the model speed response, $\tilde{\omega}$. The model response is compared with the actual speed response $\omega$, and the difference, $\omega - \tilde{\omega}$, is used as the input signal to the IMC controller, $G_c^*$.

The proposed control scheme is designed in two steps:

Step 1. The DC motor model is factored as

$$\tilde{G}_m = \tilde{G}_{m+}\tilde{G}_{m-} \tag{8}$$

where $\tilde{G}_{m-}$ is the invertible part and $\tilde{G}_{m+}$ is the non-invertible part containing any time delays and right-half plane zeros. Since there are no time delays or right-half plane zeros, the approximated plant model is the invertible part.

Step 2. The IMC controller is designed thus

$$G_c^* = \frac{1}{\tilde{G}_{m-}} f \tag{9}$$

where $f$ is a low-pass filter with a steady-state gain of one and is typically of the form

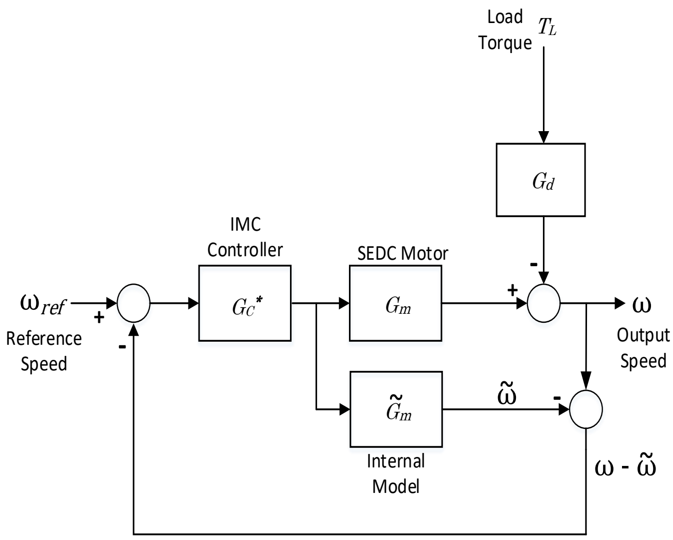


Figure 3. Internal model control structure

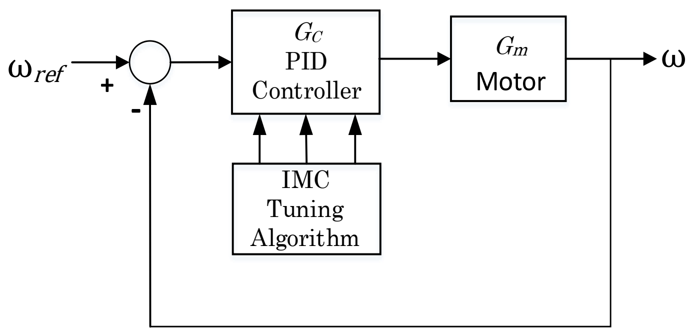


Figure 4. Equivalent IMC-tuned feedback control structure

$$f = \frac{1}{(\tau_c s + 1)^r} \quad (10)$$

where $\tau_c$ is the desired closed-loop time constant (the choice of which is a major decision in the IMC design procedure), $r$ is a positive integer parameter that will be chosen as 1 for this design. The low-pass filter $f$ helps to ensure that the resultant IMC-tuned PID controller is reasonably robust, physically realizable and stable [7,15].

Since the desired closed-loop response (transfer function) for the DC motor is a second order system with the general form

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_0} \quad (11)$$

The tuning parameter $\tau_c$ is obtained from the roots of the characteristic equation, that is, the poles of the system. These can either be real, or a complex conjugate pair. The systems damping ratio, $\zeta < 1$, which means there is some overshoot to the step response. Typically for an underdamped response the poles of the system are a complex conjugate pair ($-\sigma \pm j\omega$), hence $\tau_c$ is obtained from the reciprocal of the real part of the poles [16].

$$\tau_c = \frac{1}{|Re(s)|} \quad (12)$$

The tuning parameter $\tau_c$ ensures a reasonable trade-off between performance and robustness, where a small value is ideal for performance while a large value favours robustness. This work will endeavour to find the optimal value of $\tau_c$ that will favour good performance subject to satisfactory robustness [6]. Performance for the controller, here, is defined as excellent setpoint tracking, a fast and non-oscillatory response for the DC motor speed. Comparing the two block diagrams in Figures 3 and 4, it can be seen from block diagram transformation that the feedback and IMC controllers are equivalent if $G_c$ and $G_c^*$ satisfy the relation:

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}_m} \quad (13)$$

Hence the equivalent feedback controller in Figure 4 is a function of the DC motor model and IMC tuning algorithm. The derived IMC-tuned PID controller is in the parallel form and is expressed thus:

$$K_{PID}^{parallel}(s) = k_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \quad (14)$$

$$K_{PID}^{parallel}(s) = k_p + \frac{k_i}{s} + k_d s \quad (15)$$

where $k_p$ is the proportional gain, $k_i$ is the integral gain derived from the integral time $\tau_i$, while $k_d$ is the derivative gain which is a function of the derivative time $\tau_d$. A comprehensive list of all the parameters used for the modelling and simulation are listed in Appendix.

## 4.2 Evaluation Criteria

### *4.2.1 Performance*

Performance for the controller is measured by the ability of the motor output speed to accurately follow the reference speed. Performance for the controller, for a set point change, will be quantified with the rise time, settling time, and overshoot $M_p$. This will be used to portray a response that is fast, non-oscillatory and has a zero steady-state error.

### *4.2.2 Robustness*

In this paper performance, in simple terms, is measured with respect to robustness and will be quantified with the peak of the sensitivity function $M_s$, which is expressed as:

$$M_s = max_\omega |S(j\omega)| = \|S\|_\infty \quad (16)$$

where $\|S\|_\infty$ is the infinity norm of $S(s)$. The sensitivity and complementary sensitivity are expressed as $S(s) = 1/\left(1 + GK(s)\right)$ and $T(s) = 1 - S(s)$ respectively. With respect to robustness, $M_s$ is the inverse of the closest distance from the forward loop transfer function $L(s) = G(s)K(s)$ to the critical point, $-1$, in a Nyquist plot [6,17]. It is hence an objective of the model-based compensator to ensure that $L(s)$ is at an acceptable distance from $-1$, a measure of which is the stability margin [6]. $M_s$ is given by

$$M_s = \frac{1}{minimum\ radius\ (r)\ of\ circle\ centred\ at - 1\ touching\ L(j\omega)} \quad (17)$$

Typically, a small value of $M_s$ is desired to ensure robustness. The larger the value of $M_s$ indicates a poor robustness. Reference [14] recommends a good value for $M_s$ to be around 1.6 and should not exceed 2, while [18] recommends a value within the range 1.4-1.8.

## 5. SIMULATION RESULTS AND DISCUSSION

To illustrate the relationship between the performance and robustness of the proposed IMC-PID controller, a separately excited DC motor is simulated with the following parameters: $V_a - 120$ V, $\omega_{rated} = 1500$ rpm, $J_a = 0.02365\ \mathrm{kg.m^2}$, $R_a = 1.5\ \Omega$, $L_a = 0.2$ H, $K = 0.67609\ \mathrm{Nm.A^{-1}}$, $B = 0.002387\ \mathrm{Nm.s.rad^{-1}}$ and $\omega_{ref} = 1200$ rpm.

The open-loop response of the DC motor to a 1200 rpm step reference speed is shown in Figure 6. It is assumed that there is no load torque disturbance to the system. As expected, it is oscillatory with a large steady-state error. The closed-loop response of the SEDCM to the reference speed with the conventional PID ($k_p = 1.2, k_i = 7.5, k_d = 0.048$) controller compared with IMC-tuned PID controller for various values of $\tau_c$ is shown in Figure 7. It can be seen that the IMC-tuned feedback controller gives a better performance that is fast and non-oscillatory. The IMC-tuned PID controller unlike the conventional ensures that the DC motor step response is properly damped, by improving the damping ratio ($\zeta = 1$), leading to an elimination of the overshoot (non-oscillatory response). A comparison of the closed-loop speed response of the IMC-tuned PID controller for different values of $\tau_c$ is shown in Figure 8.

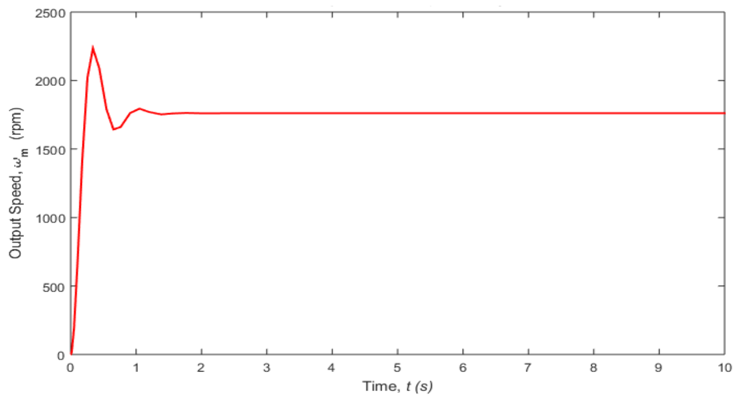


Figure 6. Open-loop response of the SEDCM to a reference speed of 1200 rpm

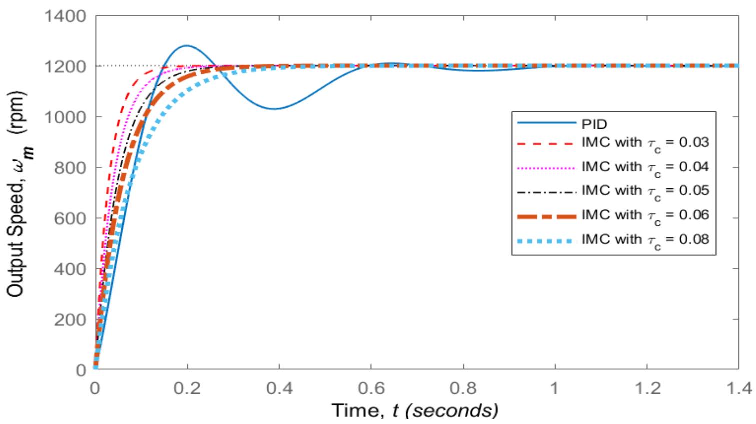


Figure 7. Closed-loop response of the SEDCM with the IMC-PID and PID controllers

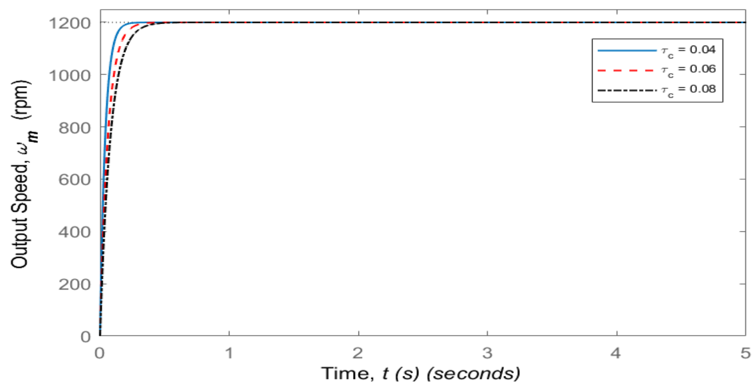


Figure 8. Closed-loop response of the IMC-PID controller for various values of $\tau_c$

The Nyquist plots of the forward loop transfer function for the IMC-PID controller (with $\tau_c = 0.06$) and conventional PID is shown in Figures 9 and 10. It is observed that the stability margin and the robustness of the DC motor is considerably improved, see Table 1 for a detailed comparison of the performance for various values of $\tau_c$.

The derived IMC settings and summary of their performance characteristics for various values of $\tau_c$ as a tuning parameter is given in Table 1. It can be seen that the model-based feedback controller produces a faster response and improved performance compared to the PID. It can also be seen that as the choice of $\tau_c$ increases the robustness of the system is improved at the expense of performance and vice versa.

Hence, the optimal controller setting that will ensure tight control is the IMC-PID with $\tau_c = 0.06$. With this controller settings the robustness of the system is improved and close to the recommended value, $M_s = 1.6$, at the same instance the performance of the system with respect to the specified criteria is very good. This IMC-tuned PID controller with $\tau_c = 0.06$ is the recommended setting that will ensure tight control for the DC motor.

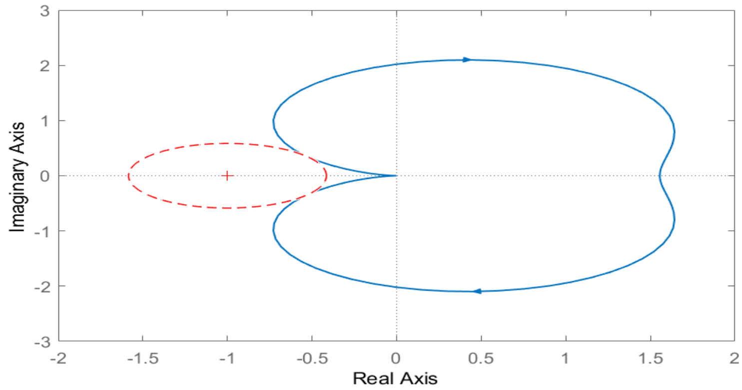


Figure 9. Nyquist plot of the IMC-PID controller with $\tau_c = 0.06$

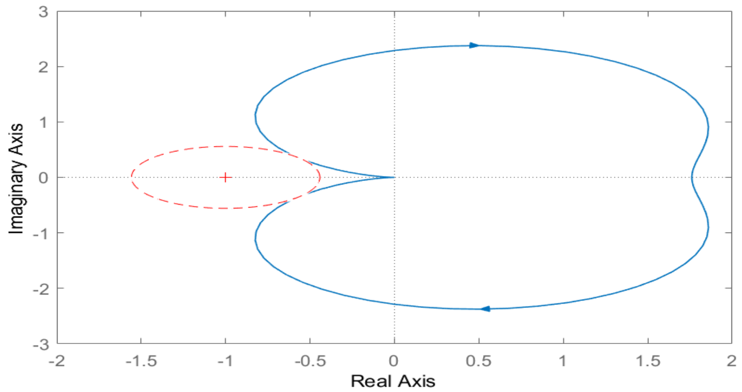


Figure 10. Nyquist plot of the PID controller for the control of the DC motor speed

Table 1. Derived IMC-PID controller settings and performance summary with $\tau_c$ as a tuning parameter

| Controller | $\tau_c$ (s) | $k_p$ | $k_i$ | $k_d$ | Rise Time (s) | Settling Time (s) | $M_p$ (%) | $e_{ss}$ (rpm) | Minimum Distance Between $L(s)$ and critical point, $r$ | $M_s = \frac{1}{r}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| PID | - | 1.2 | 7.5 | 0.048 | 0.110 | 0.555 | 6.58 | 0 | 0.558 | 1.79 |
| PID-IMC | 0.03 | 1.770 | 22.700 | 0.233 | 0.066 | 0.117 | 0.00 | 0 | 0.474 | 2.11 |
| PID-IMC | 0.04 | 1.330 | 17.030 | 0.175 | 0.088 | 0.157 | 0.00 | 0 | 0.534 | 1.87 |
| PID-IMC | 0.05 | 1.060 | 13.620 | 0.1398 | 0.090 | 0.196 | 0.00 | 0 | 0.585 | 1.71 |
| PID-IMC | 0.06 | 0.885 | 11.350 | 0.1165 | 0.132 | 0.235 | 0.00 | 0 | 0.625 | 1.60 |
| PID-IMC | 0.08 | 0.664 | 8.514 | 0.0874 | 0.176 | 0.313 | 0.00 | 0 | 0.686 | 1.46 |

## 6. CONCLUSION

In this paper an optimal setting for the IMC-tuned feedback scheme, based on the closed-loop time constant, for the control of the speed of the separately-excited DC motor drive is presented. This optimal controller setting allows the perfect trade-off between performance and robustness. The result of this research validates the need to strike a balance while tuning the model-based controller and hence, find an optimal control point for the machine such that there is a perfect trade-off between performance and robustness. This model-based feedback action brings us close to providing a solution to this control challenge. However, because the controller is based on an approximate model of the motor there could be limitations of model inaccuracy (plant-model mismatch) and external disturbances for the model-based controller. Hence, further research should be devoted to identifying a more accurate nonlinear model of the DC motor, and combining the model-based controller with a scheme that will address disturbances effectively. Finally, the paper concludes with a simulation of the proposed strategy using real DC motor parameters to show its practicability.

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## APPENDIX

| Parameter | Description |
|---|---|
| $R_a$ | Armature resistance in ohms |
| $E_b$ | Back EMF in volts |
| $L_a$ | Armature inductance in Henry |
| $L_f$ | Field inductance in Henry |
| $V_f$ | Field voltage in volts |
| $K$ | Torque to back EMF proportionality constant in $\mathrm{Nm.A^{-1}}$ |
| $k_p$ | Flux to armature current proportionality constant |
| $k_i$ | Motor torque to armature current proportionality constant |
| $k_d$ | Back EMF to speed proportionality constant |
| $B$ | Motor viscosity in $\mathrm{Nm.s.rad^{-1}}$ |
| $J$ | Motor moment of inertia or reflected inertia in $\mathrm{kg.m^2}$ |

| | |
|---|---|
| $T_L$ | Load torque in N-m |
| $T_e$ | Development electrical torque in N-m |
| $V_a$ | Armature voltage in volts |
| $\omega$ | Actual motor speed response in rpm |
| $\omega_{ref}$ | Reference speed input in rpm |
| $\widetilde{\omega}$ | Assumed model speed response in rpm |
| $i_f$ | Field current in A |
| $i_a$ | Armature current in A |
| $t_s$ | Settling time of response in seconds |
| $t_r$ | Rise time of response in seconds |
| $M_p$ | Peak overshoot in % |
| $M_s$ | Stability margin (Peak of sensitivity function) |
| $r$ | Radius of circle centered at -1 |
| $S(s)$ | Sensitivity function |
| $G_c$ | Equivalent feedback controller (IMC-tuned PID controller) |
| $G_m$ | Transfer function relating actual motor speed to armature voltage |
| $G_d$ | Transfer function relating motor speed and load torque |
| $G_c{}^*$ | IMC controller |
| $\tilde{G}_m$ | Approximate model of the SEDCM |
| $K_{PID}^{parallel}$ | Parallel form of a derived IMC-tuned PID controller |
| $\tau_c$ | Desired closed-loop time constant in secs |
| $\tilde{G}_{m+}$ | Non-invertible part of approximate SEDCM model |
| $\tilde{G}_{m-}$ | Invertible part of approximate SEDCM model |
| $f$ | Low-pass filter |
| $\zeta$ | Damping ratio |
| $\sigma$ | Real part of pole of the system |
| $e_{ss}$ | Steady-state error in rpm |
| $L(s)$ | Loop gain |