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Allocating Quay in Optimal Layout Design for Mixed Queuing Systems in Marine Vessels Using a Quadratic Programming Formulation

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Abstract: In this paper, we use the dataset of a marine berthing structure per hour to construct the objective and linear constraint functions from the combination of a simple and non-preemptive queueing model, and we determine the optimal design of vessels and layout of quay through the solutions of quadratic programming problems. A methodology is proposed to formulate the quadratic programming problems from the input and output data of the mixed queuing model as the constraints on the characteristics of logistic demand, service time, waiting time, vessels, and layout of service quay for minimum utilization cost. This approach is applied to the port authority of Kribi in the south region of Cameroon to examine the more precise optimal solution with nonlinear complexities in the programs of discrete computer simulations. With adaptive experimental data of the modelling of berths as mixed models, the results provide the value of the most optimal solution projected in the port of Kribi compared to the solutions of integer and traditional linear programming problems. This study delivers the optimal number of vessel types and the associated layout in order of importance among the vessel types which can be safely moored for operations.

Keywords: Berthing service; Linear optimization; Quadratic optimization; Queuing model; Port.

1. INTRODUCTION

In recent years, the solutions to complex problems of quay allocation and layout design have led to many changes due to multiple conflictual activities observed in certain port operations. This significantly impacts the port performance and the optimal utilization of berthing structures. The development of optimization systems becomes crucial for port authorities due to the difficulty of handling new constraints and risks in the logistic queuing models, such as unexpected collisions, accidents, and sanitary, economic, and security crises.

During World War II, Danzig [1] used linear programming to face the problems of allocation and limited resources in fighter airplanes, radars, and submarines. The most popular method to solve linear programming problems is the simplex method developed by Danzig and generalized very early by Wolfe [2], before the appearance of other methods for solving quadratic programming problems [3]. This important class of problems remains the best-behaved nonlinear programming problems in complex situations. It occurs, for example, in optimal strategy finding for production plans, construction, sales, buying, and distribution in manufacturing firms, communication networking industries, and the routing of aircraft and ships. Over the years, queuing models have been shown to be efficient in evaluating the optimal number of facilities within a logistic system. Huang and Wu [4] estimated the initial number of berths in a port system based on cost function. The description of the queueing system is based on Kendall's notation $A\ /\ B\ /\ m\ /\ K\ /\ n\ /\ D$, where A is the distribution function of the inter-arrival times, B is the distribution function of the service times, m is the number of servers, K is the capacity of the system, n is the population size or the number of sources of customers and D is the service discipline [5]. Markovian or memoryless systems, denoted by M, are exponentially distributed random variables. Many workers had carried out an optimal number of berths for a port's queuing system. In evaluating the number of servers at a logistic system, infinite and finite sources of queueing models were designed: Anonkye *et al.* [6] studied the vehicular traffic at a signalized intersection in the Kumasi-Ashanti Region of Ghana using the $M/M/1/\infty$ queueing model; Thiagaraj and Seshaiah [7] designed a queuing model applied to airport design to characterize delays in the terminal area using the interpolated queueing model; With the system servers denoted by Z, Toshiba Sheikh *et al.* [8] used the idea of first-come, first-served (FCFS) and converted the $\mathrm{M}/\mathrm{M}/Z/\infty$: FCFS queue model into an $M/M/1/\infty$: FCFS queue model in order to know which was more efficient; Jhala and Bhathawala [9] applied queuing theory to airport problems and analyzed the queueing characteristics of the bank using a Multi-server queueing model; Wang [10] studied the applications of the queueing theory in characterizing and optimizing the passenger flow at airport security at the Chicago O'Hare airport using an asynchronous multiple $M/M/s$ queue model.

Recently, Nalif *et al*. [11] improved the port facilities for marine pilot activities, and Adan *et al*. [12] have used functional equations with multiple recursive terms to describe queuing systems. However, one of the most used multi-criteria analytic methods designers and planners use is the Analytic Hierarchy Process [13]. This analysis method has been considered as an effective tool for dealing with complex decision-making, and aiding decision-makers in setting priorities, and making the best decision [14]. Previously, the problems of berth allocation were described by the mixed integer programming formulation [15, 16], while linear programming was used to handle queuing control problems [17]. The construction of the cost function was conducted in [18] to find the optimal value of the service rate using the quadratic Fit search method. Other methodologies were continuously used in [19]-[25] to optimize the parameters of queuing systems with the impact on an important number of applications [26]-[30]. More intensively, the act of optimizing the queuing systems is also studied in [31, 32], and the performance of their solution methodology is demonstrated in automobiles, vehicles, and networks. More recently, the adaptive solutions of berth allocation problems have been presented in [33]-[36] among the previous solutions approach of the mixed integer programming models with different environmental conditions and systems data.

In the South Region of Cameroon, the planning of the port facilities of Kribi takes into account the constraints on the nature of port operations, the type of infrastructures, the social and economic environment, and the functioning of the berthing structure characteristic of ship companies, port operators, climate change, and maritime security. Therefore, it is not easy to optimize the parameters of queuing models to allocate quay and design the optimal layout from other ideal conditions in port operations. To handle conflictual situations where particular deviations and interactions are observed between vessels, we formulate quadratic optimization problems from the mixed n simple $M/M/1$ queuing model and a m non-preemptive $M/M/1$ queue model to design the optimal number of berths and layout construction of the service pier. A methodology is presented to construct the objective function from the percentage of usage of a particular berth. All the logistic constraints have parallel services and include the characteristics of service demand, service time, waiting time, types of vessels, dimensions of vessels, and dimensions of berthing service. This approach is applied to the case of Kribi deep sea port with adaptive experimental data, and the well-behaved optimal solutions are evaluated and compared with the solution obtained from integer linear and traditional linear programming. The results present the most optimal solution with the associated layout and prioritize two berths designed for in-board military vessels and one berth designed for a floating dock in the Port Authority of Kribi for minimum utilization cost.

The remainder of this paper is organized as follows. In Section 2, the materials of the mixed queuing model are defined, and a methodology is proposed to minimize the utilization cost of a class of quadratic programming problems in the optimal layout design applications. In the same section, the statement of quadratic optimization problems is formulated based on the characteristics of design materials and hypotheses of the simple and non-preemptive queuing model. In Section 3, this approach is applied to the port authority of Kribi, and the results are presented to show the optimal solution corresponding to minimum utilization cost and optimal layout design of the quay. A short discussion of this approach is presented in Section 4, and we summarize this case study in Section 5 with some perspectives.

2. MATERIALS AND METHODS OF THE MIXED QUEUING MODEL

We describe the logistic system through a combination of n simple $M/M/1$ model and m non-preemptive $M/M/1$ queue model to formulate the quadratic optimization problem of allocation of the service quay, and for the optimal layout design in the port authority of Kribi. Queuing analysis is equally helpful for estimating capacity requirements and managing demand for any system in which the timing of service needs is random. It is essential to predict congestion levels or determine how much capacity is needed to achieve some desired level of performance, which can be easily done with the help of a queuing model. A port berthing system is described as an $M/M/n$ queue where the service is provided by servers operating independently of each other with parallel services and the interest in the distribution of first service completion, such that waiting only occurs when the number of customers k in the system becomes greater than or equal to the number of servers n. In reality, individual nodes of n parallel $M/M/n$ queues do not always work independently. Since a port system can be simulated as n parallel $M/M/1$ queues working in parallel, it can be viewed as a tandem queue, that is, when several simple $M/M/1$ queuing systems considered as nodes are connected in serial to each other. Thus, at each node, the arrival process is a Poisson process with parameter λ, and the nodes operate independently. Hence, if the service times have a parameter μ_i at the ith node, then introducing traffic intensity $\rho_i = \lambda/\mu_i$, all the performance measures for a given node could be calculated. Therefore, it is up to the designer to capture the relations between the different nodes as accurately as possible. By denoting the steady state distribution as P_k in an $M/M/n$ queue model, its expression is given by:

$$P_k = \begin{cases} P_0 \frac{\rho^k}{k!} & , for\ k \le n \\ P_0 \frac{a^k n^n}{n!} & , for\ k > n \end{cases} ,(1) \tag{1}$$

where $P_0 = \left(1 + \sum_{k=1}^{n-1} \frac{\rho^k}{k!} + \sum_{k=n}^{\infty} \frac{\rho^k}{n!} \frac{1}{n^{k-n}}\right)^{-1}$ and $a = \frac{\lambda}{n\mu} = \frac{\rho}{n} < 1$ is the utilization of a given server. The performance indexes are obtained for this model from the Erlang's formula denoted by $C(n, \lambda/\mu)$, and the probability of waiting P_w is defined as follows:

$$P_w = \frac{\frac{\rho^k}{k!}\frac{n}{n-\rho}}{\sum_{k=0}^{n-1}\frac{\rho^k}{k!} + \frac{\rho^n}{n!}\frac{n}{n-\rho}} = C(n,\rho) \tag{2}$$

and the waiting time W_t is given by:

$$W_t = P_w \frac{1}{\mu(n-\rho)} \tag{3}$$

The utilization index of the model is given for a single server by:

$$U_s = \frac{\lambda}{n\mu} = \frac{\rho}{n} \tag{4}$$

and the total utilization by equation:

$$U = nU_s. \tag{5}$$

The average number of ships in the queue is given by:

$$N_q = \lambda W_t \tag{6}$$

while the average number of ships in the system is given by:

$$N_p = \rho + \frac{\rho}{n-\rho} P_w \tag{7}$$

The cost index denoted as CI_i is given by:

$$CI_i = \frac{C_s\lambda_i\left(W_{ti}+\frac{1}{\mu}\right)}{C_s\lambda_i\left(W_{ti}+\frac{1}{\mu}\right)+C_b n(1-\rho)} \tag{8}$$

where C_s is the cost of shipping per unit of time, and C_b is the cost of berth per unit of time.

A port system simulated as an $M/M/n$ queuing system is done under the following hypotheses:

(a) There are n servers of the same type in the system working in parallel with each other.
(b) The service rate is not identical for all the customers, except for customers of the same type (company). However, an average service rate is evaluated, which is what is then used in the simulation model.
(c) Just one design vessel is considered, given that all the servers are identical.
(d) Evaluating the number of servers.

The initial number of servers (denoted by N_0 as a function of CI) is evaluated with the use of Equation (8) as follows:

$$N_0 = \alpha + \beta\sqrt{\alpha} \tag{9}$$

where

$$\alpha = \frac{\lambda}{\mu}$$

and

$$\beta = \begin{cases} 1.1 + 2.8\,exp\left(-2.5\sqrt{\alpha}\right), & for\ CI = 0.0 - 0.5 \\ 0.65 + 2.3\,exp\left(-2\sqrt{\alpha}\right), & for\ CI = 0.5 - 1.0 \\ 0.4 + 2.3\,exp\left(-1.8\sqrt{\alpha}\right), & for\ CI = 1.0 - 5.0 \end{cases}$$

with CI the cost index.

By plotting the curve of service rate denoted by S_r and waiting rate denoted by W_t, we can get the optimal service rate corresponding to the wait time given by the expected level of service standard (LOS) and calculate the number of servers as:

$$n = \frac{initial\ S_r}{read\ S_r}. \tag{10}$$

The optimal number of servers is given by:

$$n_{opt} = min(n, N_0). \tag{11}$$

A simple $M/M/1$ queue model is characterized mainly by the arrival rate λ and the service rate μ. The evaluation indexes for this system are given with the equations above for $n = 1$. A port, modeled as n parallel $M/M/1$ queue models include the following hypotheses:

(a) The different servers all have different service rates, which are a function of the type of customer arriving at the server.
(b) Each group of customers is serviced at a server specific to them.
(c) Each type of ship can be moored at only one particular berth.
(d) There is consequently design vessels considered.

However, this is not exactly what happens in reality of port operations where ships of a group of categories can use the same berth while some berths can only be used by one type of ships. As such, we came up with a model which we named the mixed model.

2.1 Non-Preemptive $M/M/1$ Model

For this model, the mean response of a non-preemptive priority $M/M/1$ model is obtained by:

$$\mathbb{E}(T_1) = \mathbb{E}(N_1)\frac{1}{\mu} + \frac{1}{\mu} + \rho_2\frac{1}{\mu}. \tag{12}$$

The mean response time for customers of class 1 by:

$$\mathbb{E}(T_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1}. \tag{13}$$

The mean number of customers of class 1 by:

$$\mathbb{E}(N_1) = \frac{(1+\rho_2)\rho_1}{1-\rho_1}. \tag{14}$$

The mean response time for customers of class 2 by:

$$\mathbb{E}(N_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))\rho_2}{(1-\rho_1)(1-\rho_1-\rho_2)} \tag{15}$$

The mean number of customers of class 1 by:

$$\mathbb{E}(T_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}. \tag{16}$$

The utilization of the system is given by:

$$U_s = \sum_i \rho_i 100 \qquad i = 2. \tag{17}$$

The average waiting time is given by:

$$W_{ti} = R_{ti}\rho_i \quad i = 1,2. \tag{18}$$

The average number of customers in queue is

$$N_{qi} = W_{ti}\lambda_i \ , \ i = 1,2. \tag{19}$$

In the case of a simulation using n parallel $M/M/1$ queue models, the assumptions are such that each type of ship can be moored at only one particular berth. By using the mixed model as a combination of n simple $M/M/1$ models and m non-preemptive $M/M/1$ queue models, the whole system is made of two sub-systems, each with its own sub-systems. Therefore, the total number of servers is the sum of the number of servers given by each sub-system. Among the design hypotheses given above for a simple and non-preemptive queuing model, the application of the mixed model takes into account the fact that:

(a) For every server, an evaluation is done to see if more than one vessel type can be serviced.
(b) A design vessel is considered for each server, and the queuing model applicable specified.
(c) At every server, a specific service rate is a function of the type of customer (uniquely for the simple $M/M/1$ models, and the customer class for the non-preemptive model) arriving at the server.
(d) At each server, there are two classes of customers, and the priority criteria considered is the service time.
(e) The number of design vessels considered is equal to the number of different server types.

All the symbols can be defined as follows:

k	Number of ships in the system servers
n	Number of servers when k ships are in the logistic system
μ_i	Mean service rate when $1 \leq i \leq k$ ships are in the system servers
λ	Mean arrival rate of ships in the system servers.
ρ_i	Traffic intensity when $1 \leq i \leq k$ ships are in the system servers
P_k	Probability of k ships in the system
W_t	Waiting time of k ships in the system
P_w	Probability of waiting time of k ships in the system
U_s	Utilization factor (index) for the service facility in the system of n server
N_q	Average number of ships in the queue
N_P	Average number of ships in the system

2.2 Quadratic Programming Formulation Using the Mixed Queuing Model in Marine Vessels

To optimize the logistic system as a quadratic optimization problem, we define the design variables, linear constraints, and quadratic objective functions by considering the design hypotheses of the mixed model, input, and output data of the combination of the simple and non-preemptive $M/M/1$ queuing model. Often, with the mixed model, the optimal number of berths and the design of the quay layout depend strongly on the environmental conditions, the nature of logistic problems, and system data. They cannot be chosen arbitrarily or adopted by a manager only under a study of the basic components of the mixed queuing model. Here, we formulate a logistic simulation model of quadratic programming, and the solution methodology is proposed to solve the problem of quay allocation and the layout design. Hence, we present the different parameters and functions that define the allocation of quay:

(a) The waiting time, which is representative of the amount of time that each ship will spend in a queue at the port, is gotten for each node.

(b) The service rate is a function of the number of berths or viewed as the number of simultaneously berthed ships and the service rate of one vessel.
(c) The utilization rate is the percentage of usage of a particular berth.
(d) The number of berths obtained for each node as a function of the different service rates.
(e) The width of the berthing structure is a function of the ship's dimensions, and the type of berthing considered.

Some of these are treated as variables in the design process, and certain quantities are usually fixed. The design variable space is a-dimensional Cartesian space $V \subset \mathbb{R}^n$ where each coordinate axis represents the value of the design variable $v_j, j = 1,2,\dots,n$ determined at each jth index. The behavior constraints $b_i, i = 1,2,\dots,p$ that represent physical limitations on v_j, such as availability, can be treated as p types of resources allocated among different activities v. The constant c_j may represent the utilization cost per unit of v_j. The coefficients a_{ij} represent the amount of ith resource b_i needed for 1 unit of jth activity v_j. All the activities, design variables, and constraints can be summarized in Table 1.

Table 1. Configuration plan of the proposed quadratic programming formulation in the berthing structure.

Purpose of the vessel at each node	Activity types (port berthing operations involving a $M/M/n$ queuing system with a finite set of activities, $A_1 A_2, \dots, A_p$ including activities related to dimensions of vessels)					Utilization at each node
Vessel types $(v_1, v_2, \dots . v_n)$	A_1	A_2	A_3	...	A_p	Cost per unit (%)
v_1	a_{11}	a_{21}	a_{31}	...	a_{p1}	c_1
v_2	a_{12}	a_{22}	a_{32}	...	a_{p2}	c_2
v_3	a_{13}	a_{23}	a_{33}	...	a_{p3}	c_3
v_4	a_{14}	a_{24}	a_{34}	...	a_{p4}	c_4
. . .	. . .	. . .	. . .	. . .	. . .	. . .
v_n	a_{1n}	a_{2n}	a_{3n}	...	a_{pn}	c_n
Physical limitations	b_1	b_2	b_3	...	b_p	

Table 1 facilitates the procedure statement in which all the admissible activities are taking into account to ensure the delivery of goods and services through a transport chain from one end to the other. Linearly, the programming problems with n design variables and p constraints are stated in the following form.

$$\begin{gathered} \max_{v \in X} f(v) = -\sum_{i=1}^{n} v_i \, c_i \\ \text{subject to } \sum_{i=1}^{n} a_{ij} \, v_i \le b_j, \quad j = 1,2,\dots,p \\ v_i \ge 0, \quad i = 1,2,\dots,n \end{gathered} \tag{20}$$

From Table 1 a linear objective function is denoted by $f(v) = C^T v$ in term of overall utilization cost, a finite number of linear constraints of queues and services is represented by the equation $A^T v^T = \sum_{i=1}^{n} a_{ij} \, v_i \le b_j = B^T$,where $v = (v_1, v_2, \dots, v_n) \epsilon \mathbb{R}^n$ is the vector of unknown vessels, $C = (c_1, c_2, \dots, c_n) \, \epsilon \, \mathbb{R}^n$ is the vector of utilization cost per unit of activities $A_1, A_2, A_3, A_4, A_5, A_6, \dots, A_p$, the vector $B = (b_1, b_2, \dots, b_p) \, \epsilon \, \mathbb{R}^p$ is the design vector of queues and services limitations, and the matrix of constraints is $A = \begin{pmatrix} a_{11} & \cdots & a_{p1} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{pn} \end{pmatrix} \epsilon \mathbb{R}^{px\,n}$. To consider the nonlinearity in the behavior of handling mixed vessel types in the real word scenarios, we represent the term $g(v) = \frac{1}{2} v^T H \, v$ as the quadratic term of the objective function where $H = \begin{pmatrix} \frac{\partial^2 g}{\partial {v_1}^2} & \cdots & \frac{\partial g}{\partial v_1 \partial v_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g}{\partial v_n \partial v_1} & \cdots & \frac{\partial^2 g}{\partial {v_n}^2} \end{pmatrix} \epsilon \mathbb{R}^{p\,x\,n}$ is a symmetric positive-definite matrix. Now, we consider the overall objective function denoted by $Q(v) = f(v) + g(v)$ and the quadratic programming problem with linear operating service stated as:

$$\begin{gathered} \max_{v \in X} \; -Q(v) \\ \text{subject to } \; A^T v^T \le B^T \\ v \ge 0 \end{gathered} \tag{21}$$

To find the optimal design, we solve the optimization problem stated in Equation (21) using Matlab optimization tools and examine the optimal solutions of computer simulations for the port authority of Kribi in Cameroon. To find the optimal design, we solve the formulated quadratic optimization problem using Matlab optimization tools and examine the optimal solutions of computer simulations for the port authority of Kribi in Cameroon through the flowchart of Figure 1.

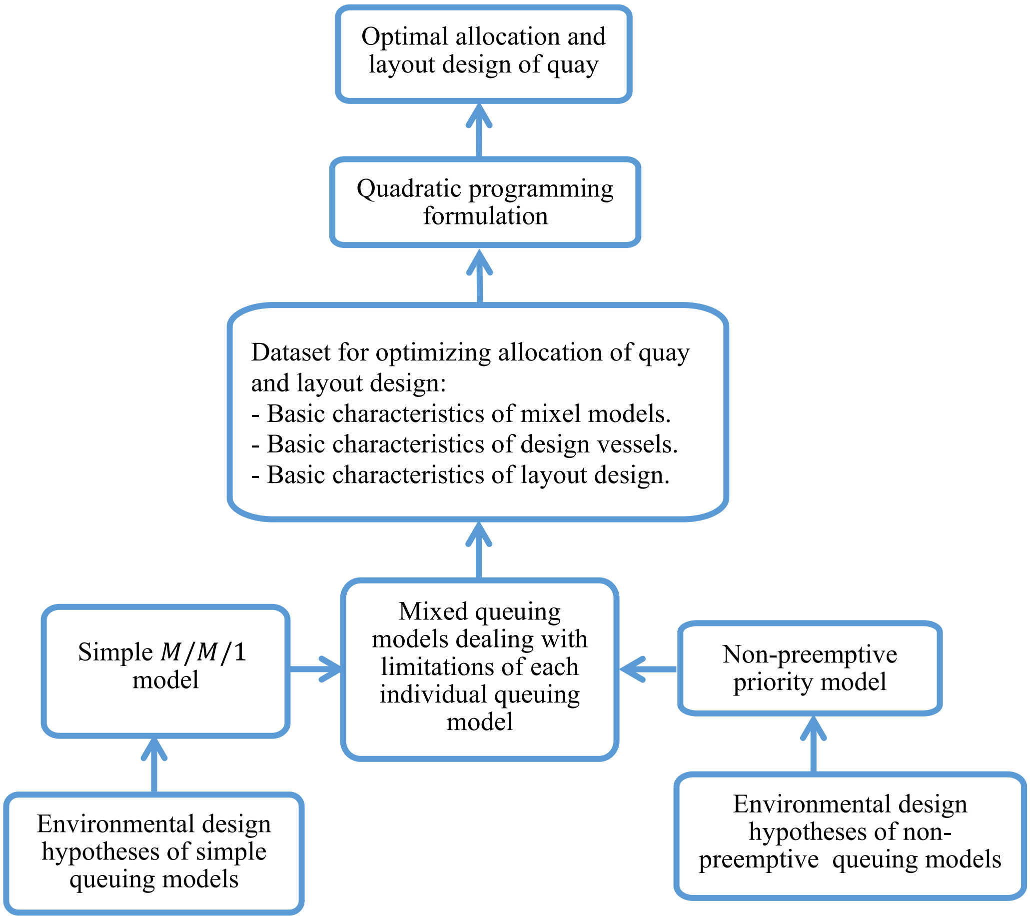


Figure 1. Flowchart for the proposed logistic simulation model of quadratic programming formulation.

3. APPLICATION OF THE QUADRATIC PROGRAMMING FORMULATION AND RESULTS

The following section illustrates the quadratic programming problem formulated from the combination dataset between the simple and $\boldsymbol{M/M/1}$ non-preemptive queuing model for the port berthing structure of Kribi, a service quay. The Kribi deep seaport, situated in Mboro, 30 km south to the city of Kribi in the South Region of Cameroon, was designed and constructed by China Harbor Engineering Corporation (CHEC) for large-scale commercial purposes constituted the following berthing infrastructures in its first phase: a 70,00 DWT capacity container terminal, a 40,000 DWT capacity multipurpose terminal, and a 61 m transition for Ro-Ro vessels. During the transition of the project from the construction phase to the exploitation phase, the acute need for a service quay where service boats could be safely moored was discovered, and the creation of the seaport has triggered the migration of shipping companies, logistic companies, and many others to the Kribi industrial zone. Some of these companies have offshore stations and, therefore, always require services for crew change, supply, and maintenance of these offshore platforms. As such, the need for the port authority of Kribi to remain autonomous by meeting the demands of these companies and ensuring the efficient daily function of port operations via the use of its tugboats and pilot boats keeps growing. The second aspect is the structural design. For the case of the Kribi deep-sea port harbor, with phase I in exploitation since March 2018 and phase II in construction since December 2019, operations to be safely hosted by the designed service quay include but are not limited to the national marine, offshore companies for operations like crew change and offshore logistic supply, the port authority for the mooring of pilot and tugboats.

Logistic information is obtained from the port captaincy for every ship type characterized in Table 2. This information specifies the demand which is viewed as the projected number of ships of this type expected to be at the port simultaneously. Also, the average service time is viewed as the amount of time spent moored by a ship at the berth to carry out its operations. From logistic data presented in Table 2, we can distinguish the following nodes:

(a) Node 1 is the berth designed for in-board military vessels or other vessels of this dimension only, and it is modelled as a simple $M/M/1$ model with the calculated number of servers N_1.

(b) Node 2 is the berth designed primarily for supply boats of dimensions (m) of 100 × 17 × 7 serving customers of type 4 and is modelled as an $M/M/1$ priority model with two class of customers and the calculated number of servers $\boldsymbol{N_2}$.

(c) Node 3 is the berth designed for a floating dock of dimensions (m) of 70 × 17 × 10 serving, when idle, boats of type 4 as well, and is therefore also modeled as a priority $M/M/1$ queue with two classes of customers, and the calculated number of servers N_3.

(d) Node 4 is the berth designed primarily for BP Boats of dimensions (m) of $33 \times 13 \times 4$ exclusively and existing two tugboats for the running of port activities, with two servers provided initially for these boats, and modeled as a $M/M/1$ queue with the calculated number of servers $N_4 + 2$.

The following data of waiting time are obtained from the system optimization modeling of the design materials of an $M/M/n$ queuing model.

Table 2. Nature of logistic demand parameters, vessel characteristics and allowable resources at the Kribi deep seaport.

Vessel type (node)	Demand	Mean service time per vessel (hour)	Waiting time (hour)	DWT (ton)	Length of berth (m)	Dimensions of vessel (m)			Purpose of the vessel
						Length	Beam	Draft	
In-board military vessels	02	16	20	10	24	10	/	3	Security on the waterfront
BP AHTS boats	02	24	3	50-120	125	12-100	5-17	4-7	Logistic support, Supply and refuelling
Floating dock	01	72	1.5	700	75.25	70	17	4	National marine
BP boats	02	2	2.5	80	46.2	33	13	6	Moor tugboats, convey pilots, etc.
Maximum activities, times, materials, and dimensions available per week	5	105	48	800	130	90	20	10	Limitations of available activities, materials, dimension

3.2 Dataset of Non-Preemptive Priority $M/M/1$ Queuing Model Applied in the Design of the Number of Port Berths

A port system can be modeled as a parallel non-preemptive $M/M/1$ queue model. In this model, two customer classes are considered at each server. Class 1 customers are considered priority over those of class 2 by the service duration criteria. This is only possible in the case where class 1 and 2 customers can use the same server. In that case, when customers of class 1 arrive, and a customer of class 2 is being serviced, the service is not preempted. However, if class 1 and 2 customers arrive at the same time, customers of class 1 will be served in priority to the customers of class 2. For each node in Figure 2, the waiting time is the amount of time each ship will spend in a queue at the port. The horizontal axis of Figure 2 represents the service rate, a function of the number of berths and the service rate of one vessel. From the waiting time graphs of nodes 2 and 3, we realize that the wait time never goes beyond 3 hours as it is also observed on node 4. Vessels rarely have to wait beyond the acceptable time of 3 hours. The utilization rate is characterized by the percentage of usage of a particular berth; thus, for each node, the variation of different service rates to the percentage of usage is plotted in Figure 3. Also, the horizontal axis represents the eleven different service rates taken into account during the evaluation of the waiting time for each node. We also observe on the utilization graph of Figure 3 that node 4 is rarely congested. As such, node 4 is necessary as an individual node and is consequently deleted. The priority for nodes 2 and 3 changes, and the criteria become: in an event wherein customers arrive simultaneously, the priority customer is he who has the shortest service time. Thus, we modified the waiting time and utilization for nodes 2 and 3 in Figure 3(b).

The objective is not only to find the optimal number of berths obtained for each node as a function of the different service rates and initial number of servers due to the limitations on the various activities in the port berthing structure of Kribi. To design the layout of quay we solve the proposed quadratic optimization model to determine the optimal number of vessel types while satisfying the design constraints of basic component of a simple and $M/M/1$ non-preemptive queuing model for minimum utilization cost. With the computer simulation, the solutions of integer, linear and quadratic programs are compared in Tables 3, 4 and 5 for the problem of allocation of quay with the specific dataset of the mixed queuing model.

Integer programming based on the Branch-Bound algorithm takes integral values, not real ones. For the layout construction, the optimal solution of integer linear programs shows that only a military vessel for security on the waterfront and a vessel of the floating dock of national marine can be safely moored for minimum utilization cost. The branch and bound method take much longer to solve integer programs with eleven iterations of the simplex method. Depending on the port situation, the number of berths to be allocated to each activity in the port of Kribi is treated as an integer variable. Consequently, the number of berths which minimizes the cost of utilization is only two. By the nature of environmental constraints and system data, the decision variables are integral, and the optimal solution of integer linear programming does not allow for the construction of enough berths if the rate of ship arrival changes over time. Regarding the optimal layout design, this solution indicates that many option criteria are not optimized during the evaluations using the multi-criteria analytic hierarchy process, including length, accessibility, port construction, port activities, and flexibility, capacity, and ship type. Table 4 shows that the integer linear programs can be solved as linear programs for the real case of a problem with continuous decision variables.

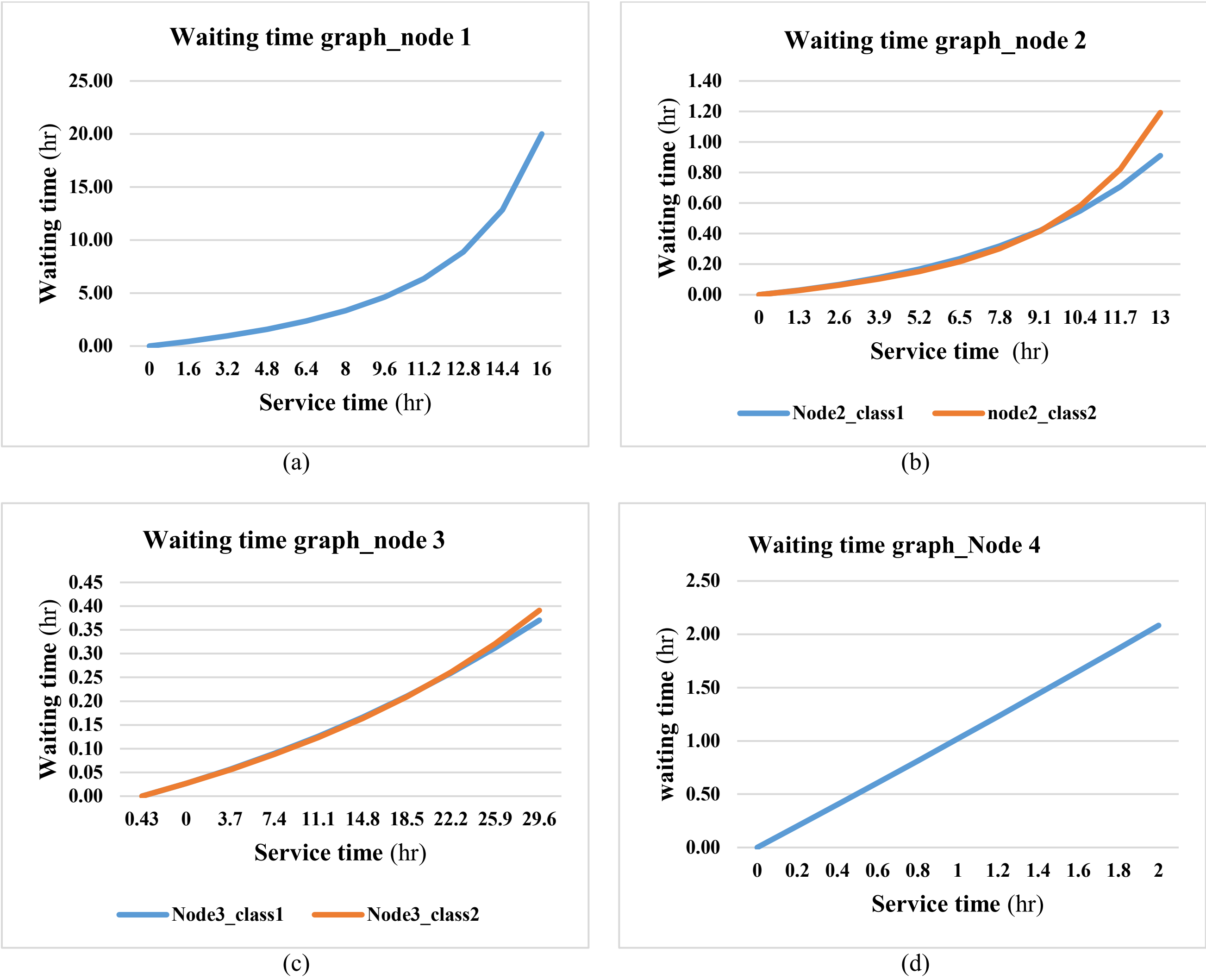
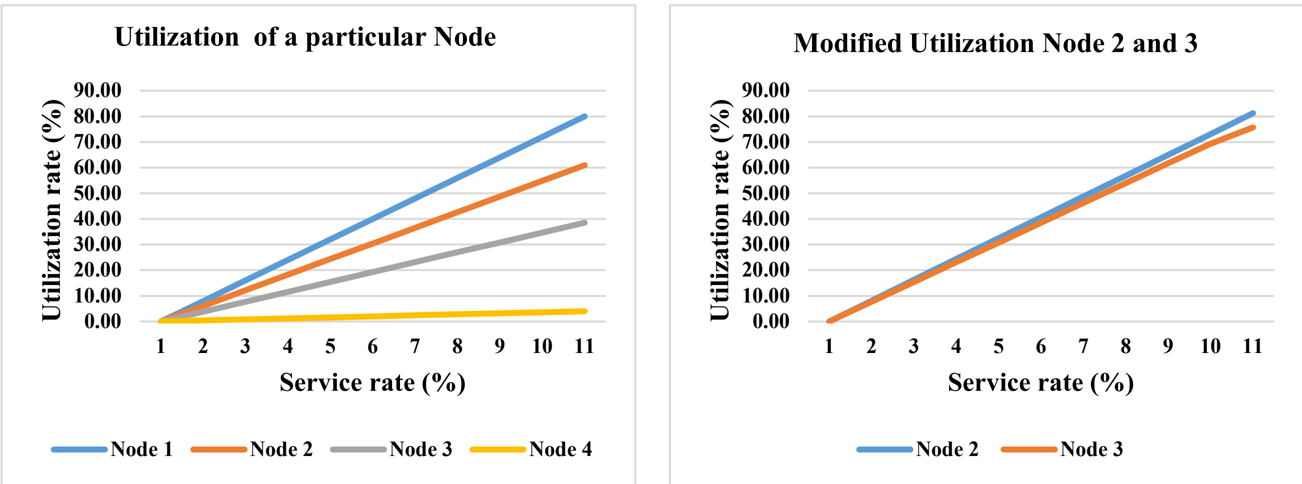


Figure 2. Nodes waiting time: (a) Node 1; (b) Node 2; (**c**) Node 3; (d) Node 4.



(a) (b)

Figure 3. Utilization rates at nodes for: (a) Nodes 1, 2, 3 and 4; (b) Modified Utilization at node 2 and 3

Linear programming based on the simplex algorithm takes real values and shows the improvement of the optimal solution compared to integer linear programming. This solution integrates the fact that the rate of ships' arrival can change, and enough berths can be constructed. Under the same design constraints, the optimal solution for integer linear programming problems is different from the optimal solution for real linear programming problems. The utilization cost of the integer linear programming technique is smaller than the utilization cost of the real linear programming technique. With the number of iterations equal to three, the linear time of the simplex method is always faster than the branch-bound and active set method. The optimal solution of linear programs is among the candidates for the optimal layout design. Only two military vessels are needed for security on the waterfront, and a vessel on the floating dock of a national marine can be safely moored for minimum utilization cost. Table 5 shows that the linear optimization problems that require an integer and real solution can be solved as quadratic programming problems.

The quadratic programs deal with the interactions between the vessel types and nonlinearities. A set of Hessian matrices $(H_i)_{1 \le i \le 7}$ is tested based on port activities of several vessel types and evaluated to minimize the utilization cost. Compared to the solution of linear programming, two military vessels for security on the waterfront and a vessel of the floating dock of the national marine can be safely moored for a finite number of minimum utilization costs. Under the same environmental conditions, the linear constraints of the proposed quadratic optimization model are satisfied at the same optimum point as the linear optimization model for a smaller minimum utilization cost. For optimal layout construction, quadratic programming is the most optimal solution for minimum utilization cost.

Table 3. Solutions of integer linear programming problems for quay allocation using the dataset of mixed queuing model.

<table>
<tr><th>Linear part of the cost function $f(v)$= $C^T v$ with the changes in utilization rates</th><th>Minimum value of utilization cost</th></tr>
<tr><td>$85v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>125</td></tr>
<tr><td>$80v_1 + 60v_2 + 50v_3 + 5v_4$</td><td>130</td></tr>
<tr><td>$80v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>120</td></tr>
<tr><td>$80v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>120</td></tr>
<tr><td>$80v_1 + 75v_2 + 30v_3 + 5v_4$</td><td>110</td></tr>
<tr><td>$80v_1 + 75v_2 + 30v_3 + 10v_4$</td><td>110</td></tr>
<tr><td>$75v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>115</td></tr>
<tr><td>$70v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>110</td></tr>
<tr><td>$70v_1 + 50v_2 + 30v_3 + 10v_4$</td><td>100</td></tr>
<tr><td>$60v_1 + 50v_2 + 20v_3 + 5v_4$</td><td>80</td></tr>
</table>

Method: Branch and bound strategy, number of iterations: 12, $v_1 = 1, v_2 = 0, v_3 = 1, v_4 = 0$.

Table 4. Solutions of real linear programming problems for quay allocation using the dataset of mixed queuing model.

<table>
<tr><th>Linear part of the cost function $f(v)$= $C^T v$ with the changes in utilization rates</th><th>Minimum value of utilization cost</th></tr>
<tr><td>$80v_1 + 60v_2 + 50v_3 + 5v_4$</td><td>210</td></tr>
<tr><td>$75v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>190</td></tr>
<tr><td>$70v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>180</td></tr>
<tr><td>$70v_1 + 50v_2 + 40v_3 + 20v_4$</td><td>180</td></tr>
<tr><td>$65v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>170</td></tr>
<tr><td>$60v_1 + 50v_2 + 40v_3 + 20v_4$</td><td>160</td></tr>
</table>

Method: Simplex, number of iterations: 3, $v_1 = 2, v_2 = 0, v_3 = 1, v_4 = 0$.

Table 5. Solutions of quadratic programming problems for quay allocation using the dataset of mixed queuing model.

<table>
<tr><th>Linear part of the cost function $f(v)$= $C^T v$ with the changes in utilization rates</th><th>Minimum value of utilization cost</th><th>Quadratic part of the cost function $g(v) = \frac{1}{2} v^T H v$ with the changes in matrix H</th><th>Number of iterations</th></tr>
<tr><td>$85v_1 + 75v_2 + 50v_3 + 20v_4$</td><td>219</td><td rowspan="15">$H_0 = \begin{bmatrix} 2 & 2 & 0 & 0 \\ -2 & 2 & 0 & 0 \\ -4 & 0 & 2 & 0 \\ -4 & 0 & 0 & 2 \end{bmatrix}$
$H_1 = \begin{bmatrix} 2 & 0 & -4 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -4 & 2 \end{bmatrix}$</td><td rowspan="15">4-6</td></tr>
<tr><td>$85v_1 + 75v_2 + 40v_3 + 20v_4$</td><td>209</td></tr>
<tr><td>$85v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>209</td></tr>
<tr><td>$82v_1 + 75v_2 + 50v_3 + 20v_4$</td><td>213</td></tr>
<tr><td>$80v_1 + 75v_2 + 50v_3 + 20v_4$</td><td>209</td></tr>
<tr><td>$80v_1 + 60v_2 + 50v_3 + 20v_4$</td><td>209</td></tr>
<tr><td>$80v_1 + 60v_2 + 50v_3 + 5v_4$</td><td>209</td></tr>
<tr><td>$80v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>199</td></tr>
<tr><td>$75v_1 + 60v_2 + 50v_3 + 20v_4$</td><td>199</td></tr>
<tr><td>$75v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>189</td></tr>
<tr><td>$70v_1 + 50v_2 + 50v_3 + 5v_4$</td><td>189</td></tr>
<tr><td>$70v_1 + 60v_2 + 40v_3 + 20v_4$</td><td>179</td></tr>
<tr><td>$70v_1 + 50v_2 + 40v_3 + 20v_4$</td><td>179</td></tr>
<tr><td>$65v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>169</td></tr>
<tr><td>$60v_1 + 50v_2 + 40v_3 + 5v_4$</td><td>159</td></tr>
<tr><td>$85_1 + 75v_2 + 50v_3 + 20v_4$</td><td>215</td><td rowspan="10">$H_2 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, H_3 = \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 4 & 0 & 2 \end{bmatrix},$
$H_4 = \begin{bmatrix} 2 & 4 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 4 & 2 \end{bmatrix}, H_5 = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$</td><td rowspan="10">4-8</td></tr>
<tr><td>$82v_1 + 75v_2 + 50v_3 + 20v_4$</td><td>209</td></tr>
<tr><td>$80v_1 + 60v_2 + 50v_3 + 5v_4$</td><td>205</td></tr>
<tr><td>$80v_1 + 75v_2 + 50v_3 + 20v_4$</td><td>205</td></tr>
<tr><td>$75v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>185</td></tr>
<tr><td>$80v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>195</td></tr>
<tr><td>$75v_1 + 60v_2 + 50v_3 + 20v_4$</td><td>195</td></tr>
<tr><td>$75v_1 + 60v_2 + 40v_3 + 5v_4$</td><td>185</td></tr>
<tr><td>$70v_1 + 50v_2 + 50v_3 + 5v_4$</td><td>185</td></tr>
<tr><td>$70v_1 + 60v_2 + 40v_3 + 20v_4$</td><td>175</td></tr>
</table>

$70v_1 + 50v_2 + 40v_3 + 5v_4$	175	$H_6 = \begin{bmatrix} 2 & 4 & 0 & 0 \\ 0 & 2 & 2 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 4 & 2 \end{bmatrix}, H_7 = \begin{bmatrix} 2 & 4 & -2 & -2 \\ 2 & 2 & -2 & 4 \\ 2 & -2 & 2 & -2 \\ 2 & -2 & 4 & 2 \end{bmatrix}$	
$65v_1 + 60v_2 + 40v_3 + 5v_4$	165		
$60v_1 + 50v_2 + 40v_3 + 5v_4$	155		

Method: Active set, $v_1 = 2, v_2 = 0, v_3 = 1, v_4 = 0$.

Table 6. Number of berths required for the optimal layout of service quay.

Type of vessel	Length of ship (m)	Number of berths	Clearance distance (m)	Allowance distance (m)	Length of berth (m)
In-board military vessels	70 - 80	2	20 - 50	3	24
Supply boats	100	0	10	15	125
Floating dock	70	1	/	5.25	75.25
PAK pilot boats	33	0	3.3	9.9	46.2

Table 7. Data of option ranking vector, score matrix and option ranking.

Criteria	Option 1	Option 2	Option 3	Weight vector
length	0.20	0.47	0.33	0.114
accessibility	0.26	0.26	0.47	0.114
port construction	0.07	0.36	0.57	0.159
port activities	0.20	0.35	0.45	0.205
flexibility	0.15	0.40	0.45	0.182
capacity	0.25	0.38	0.38	0.136
Ship type	0.32	0.32	0.37	0.091
Option ranking	0.19	0.36	0.44	/

Table 8. Option criteria scaling with highest degree of importance.

No	Criteria	Option 1	Option 2	Option 3
1	length	3	7	5
2	accessibility	5	5	9
3	port construction	1	5	8
4	port activities	4	7	9
5	flexibility	3	8	9
6	capacity	6	9	9
7	ship type	6	6	7

3.3 Adaptive Layout of Quay Using the Quadratic Programming Formulation for the Port of Kribi

The layout is designed based on the optimal solution of the logistic simulation model of quadratic programming. The dimensions of the berthing structure are obtained by evaluating the following parameters: the length, width, turning circle, and the berthing area. We have the minimum allowable distance between two ships when they are berthed in the same manner along a berth. It may be longitudinal and or transverse berthing (Table 6).

The structure designed is entirely independent of other port infrastructures, and the multi-criteria analysis method is applied to choose the optimal layout. This will permit direct access to the pier from the land side without necessarily passing through the port phase 2 access paths. Therefore, the optimal number of berths is 3 for minimum utilization cost. In order to choose the most optimal layout, the multi-criteria analysis method process is used to determine the best layout as a function of the requirements of port managers and operators. For this layout, operations and construction can be done without interference from other port activities and structures. It is represented by the option ranking vector, a function of the weight vector, and the matrix (Table 7).

Each option is therefore ranked for all the seven criteria presented in Table 8. The Figure 4 of the radar diagram shows that the option three covers the highest area on the plot. This is indicative of the fact that it is the best option with respect to the evaluation criteria and their respective scores. The option ranking vector introduces the results obtained from the logistic constraints and applications of the analytic hierarchy process (AHP). This indicates the ranking of each option in order of preference relative to the evaluation criteria. According to the criteria of Table 8 and their respective scores in Table 9, option 3 is the most optimal for the Kribi deep seaport service pier. Therefore, option 3 is chosen as the optimal layout for the port berthing structure of Kribi, and according to the number of vessel types, it minimizes the utilization cost. This option offers the possibility of coupled mooring of vessels, but it seems more economically favourable than other options.

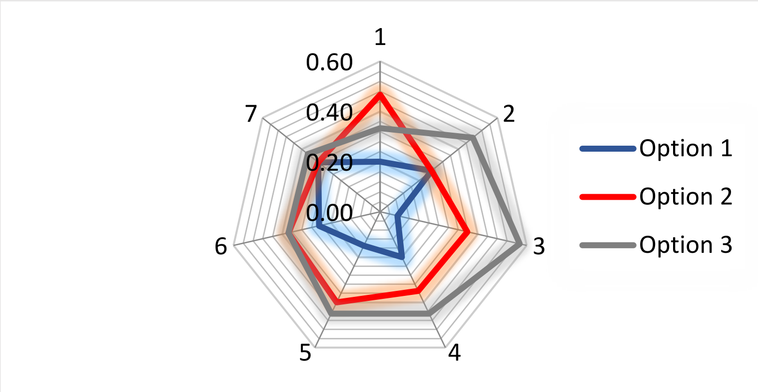


Figure 4. Radar diagram for the multi-criteria evaluation of layout options.

Table 9. Set of the option, AHP score and rank.

Option	AHP score	Rank
1	0.19	3
2	0.36	2
3	0.44	1

4. DISCUSSION

It is difficult to design the optimal layout from the input or output of queuing systems without developing an optimization strategy that considers the factors influencing the port performance at this zone. Integrating all the environmental and economic constraints related to the allocation of the quay and the construction of the layout requires an adaptive system optimization in the port authority of Kribi. Under linear constraints of the same port berth operations, the integer, linear, and quadratic programming can change the minimum utilization cost at the optimum point for the layout construction. These mathematical programs can play an important role in supporting some decisions on the utilization of port capacity to avoid any waiting time and to reduce the high cost of construction and maintenance of berths. Linear and quadratic programming formulations are alternative formulations showing that two military vessels and the floating dock are necessary and more important in finding the number of berths for minimum utilization cost and layout construction. Particularly, a vessel arriving at the Kribi deep seaport in Cameroon can be unable to berth for various reasons, such as the risks of sanitary, social, economic, and military crises. Using the branch and bound method, formulating the allocation problem of quay as an integer linear programming problem can be useful in developing a good approximation.

In many cases, the integer linear programs are much more difficult to solve with a non-convex set than convex optimization problems. The quadratic programming formulation is proposed to deal with some interactions between the vessels, including the input and output data of the combination of a simple $M/M/1$ and non-preemptive queuing model. This quadratic optimization model is not limited to additional decision variables and constraints arising from the different activities of vessels or customers. It could be reformulated and used to describe complex, large-scale mathematical programming problems of allocation under similar environmental conditions. At the same time, the nonlinear objective function is a function of number of berths in the port of Kribi and it is not restricted to a simple multi-objective form. For the designers, additional constraints of the optimization model could include not only the different vessel characteristics in dimensions and capacities, their arrival rates, their waiting times, and their determined or projected service rates, but also climate change and pandemics. Here, the nonlinear objective function represents the utilization cost of its capacity. The obtained results show that the optimal number of berths is the same using the linear and quadratic programming techniques. In contrast, the linear time method of the simplex method is faster than the quadratic time method of the active set. However, the proposed approach shows a significant improvement with the smaller minimum utilization cost under the same linear constraints for the port operations of Kribi. Thus, the value of minimum utilization cost obtained with the linear programming is greater than the one obtained with the quadratic programming due to the well-behaved quadratic functionality of the proposed strategy.

From these results, the applications of analytic hierarchy process deliver the most associated layout based on the corresponding options and criteria scaling, resulting in a radar diagram and option ranking. The most optimal solution depends on the performance of the solution methods making the design of the optimal layout of the quay more challenging. However, a framework could be developed to maximize unfavourable option criteria for the optimal design of the layout. Also, some efficient formulations could be developed to improve the presented solution methodology, including mixed integer quadratic programming problems.

5. CONCLUSION

In this work, a solution methodology has been proposed to formulate quadratic programming problems of quay allocation and layout design in mixed queuing systems, which is described as the combination of a simple and non-preemptive queueing model. The effectivity of the optimal design of layout has been measured in a port authority of Kribi to find the optimal number of vessel types that satisfy the constraints of available characteristics of the mixed model, vessel types, and layout design of the service quay. We saw that the choice of the presented logistic simulation model through the quadratic programming model could influence the well-behaved utilization cost of berthing service and, consequently, the associated layout design of available operations. In comparison to the optimal solutions of integer and linear programming problems, we found that the proposed quadratic programming formulation takes into account the nonlinear complexities of quay allocation and provides the most optimal solution for minimum utilization cost. Based on the corresponding options scaling and criteria scaling and resulting radar diagram and option ranking, these results have allowed us to verify that the choice of the proposed layout construction of an independent service is the most associated option. This approach can be applicable in many similar situations by optimizing multi-objective functions and integrating additional decision variables and specific constraints. The general problem of the port authority of Kribi is the utilization of a hundred percent of its capacity and the minimization of its costs if an important number of berths is constructed and maintained. This approach could help planners and designers to take more important decisions in complex problems of quay allocation to avoid the deviations. It could help to address more optimization problems in multiple ports, including efficient methods for large-scale mathematical programming problems. Definitively, it opens new theoretical perspectives in industrial applications of mathematical programming based on the mixed queuing models.

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DECLARATION OF CONFLICTING INTERESTS

The authors declare no potential conflicts of interest with respect to the research and publication of this article.

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