

# $C^0$ -TSDT-FEM for Free Vibration Analysis of Functionally Graded Cylindrical Shells

Lan Hoang Ton That

Faculty of Civil Engineering, HCMC University of Architecture, Vietnam

Corresponding author: [tonthathoanglan.247@gmail.com](mailto:tonthathoanglan.247@gmail.com)

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**Abstract:** In this article, natural frequencies of functionally graded (FG) cylindrical shells are calculated by applying the finite element method based on a  $C^0$ -type of the novel third-order shear deformation theory (TSDT), namely Shi's theory. In this type, only the first derivative of transverse displacements is requested after two variables are added. The material properties are changed from the top to bottom under a power-law distribution. Comparison between results from this work and the solution from other work suggests that this code is accurate and efficient.

**Keywords:** Cylindrical shell;  $C^0$ -TSDT; Finite element method; Functionally graded; Natural frequency.

## 1. INTRODUCTION

In recent decades, the finite element method (FEM) has been greatly improved to solve mechanical problems. There are different approaches based on classical FEM such as smoothed FEM, meshless method and isogeometric method. Shi's theory is one of the third-order shear deformation theories (TSDTs) with many benefits as [1], and it was used for static analysis of composite plates in a thermal environment [2]. Moreover, in [3, 4], the  $C^0$ -type of Shi's theory was applied to stiffened functionally graded (FG) plates or FG skew plates. Besides, the twice interpolation strategy can be applied to solve plate and shell structures with various advantages and without increasing the total number of degrees of freedom [5]. Moreover, some novel quadrilateral elements based on combined strains procedure or Chebyshev polynomials were presented in [6-8].

For the Shi's theory, the  $C^0$ -TSDT is firstly considered to analyze the FG cylindrical shells. It is commonly used as it gives accurate transverse shear stresses without shear correction factors. Because the quadrilateral element with four nodes is a low-order element, it needs modification of the displacement field. More specifically, the TSDT of Shi must be in a revised form with the only requirement of  $C^0$  continuity for displacement field. For the above purpose, two additional variables are considered in the displacement field respectively. Besides, several authors introduced works related to FG shells under various theories such as semi analytical three-dimensional (3D) model and two-dimensional (2D) higher-order deformation theory, as described in [9-20]. In [9], the authors investigated free vibration and dynamic instability of FG cylindrical panels subjected to combined static and periodic axial forces and in thermal environment. Theoretical formulations were based on Reddy's higher order shear deformation shell theory to account for rotary inertia and the parabolic distribution of the transverse shear strains through the panel thickness. Natural frequencies and buckling stresses of shallow shells made of FG materials were analyzed by taking into account the effects of transverse shear and normal deformations, and rotatory inertia. The modulus of elasticity of shells was assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. By using the method of power series expansion of displacement components, a set of fundamental dynamic equations of a 2D higher-order theory for rectangular FG shallow shells was derived through Hamilton's principle [10]. Based on the 3D elasticity theory, free vibration analysis of FG curved thick panels under various boundary conditions was studied in [11]. Two different models of material properties variations based on the power law distribution in terms of the volume fractions of the constituents and the exponential distribution of the material properties through the thickness were considered. A differential quadrature method in conjunction with the trigonometric functions was used to discretize the governing equations. With a continuous material properties variation assumption over the thickness of the curved panel, the differential quadrature method was efficiently used to discretize the governing equations and to implement the related boundary conditions at the top and bottom surfaces of the curved panel and in strong form.

The free vibration analysis of the FG cylindrical shell panels with and without cutouts was carried out by using the FEM based on a higher-order shear deformation theory in [12]. A higher-order theory was used to properly account for transverse shear deformation. An eight-node degenerated isoparametric shell element with nine degrees of freedom at each node was

considered. The stiffness and mass matrices were derived based on the principle of minimum potential energy. The stiffness and mass matrices of the element were evaluated by performing numerical integration using the Gaussian quadrature. The effect of the volume fraction exponent on the fundamental natural frequency of FG cylindrical shell panel without a cutout was studied for various aspect ratios and arc-length to thickness ratios. In [13, 14], a new element applied in the analysis of FG structures was introduced and the possibility of nonlinear analysis for these structures was presented.

In this article, natural frequencies of FG cylindrical shells are calculated by applying the FEM based on a  $C^0$ -type of Shi's theory. In this type, only the four-node quadrilateral element is required after two degrees of freedom per node are added. This article is divided into four sections. In Section 2, the novel quadrilateral element related to  $C^0$ -TSDT of Shi's theory is given. Some examples are presented in Section 3, and discussions are given in Section 4.

**2. THE NOVEL QUADRILATERAL ELEMENT UNDER  $C^0$ -TSDT**

An FG cylindrical shell is illustrated in Figure 1. The top and bottom faces are to be fully ceramic and metallic. The values  $V_c$  for ceramic and  $V_m$  for metal are presented in Equation (1), and the change of volume fraction under the power-law distribution is also described in Figure 1.

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \quad V_m = 1 - V_c \quad 0 \leq n \tag{1}$$

$z$  is changed from  $-h/2$  to  $h/2$ ,  $c$  and  $m$  denote the ceramic and metal constituents respectively, and  $n$  is the volume fraction gradient index. Furthermore,  $E$ ,  $\rho$ ,  $\nu$  and  $\alpha$  can be listed as

$$E(z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{2}$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{3}$$

$$\nu(z) = \nu_m + (\nu_c - \nu_m) \left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{4}$$

From the Shi's theory in [1], the displacement field ( $u, v, w$ ) was introduced as

$$u(x, y, z) = u_0(x, y) + \frac{5}{4} \left(z - \frac{4}{3h^2} z^3\right) \beta_x(x, y) + \left(\frac{1}{4} z - \frac{5}{3h^2} z^3\right) w_{0,x}(x, y) \tag{5}$$

$$v(x, y, z) = v_0(x, y) + \frac{5}{4} \left(z - \frac{4}{3h^2} z^3\right) \beta_y(x, y) + \left(\frac{1}{4} z - \frac{5}{3h^2} z^3\right) w_{0,y}(x, y) \tag{6}$$

$$w(x, y, z) = w_0(x, y) \tag{7}$$

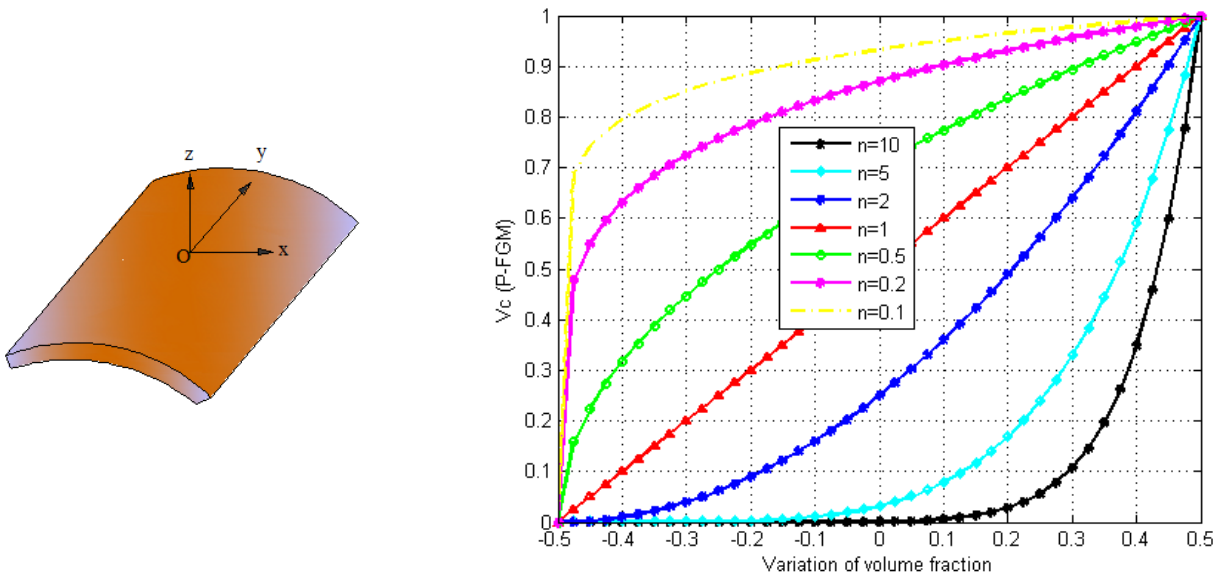


Figure 1. The FG cylindrical shell and the change of volume fraction

This field is modified in  $C^0$ -TSDT with seven unknown variables as [3]

$$u(x, y, z) = u_0(x, y) + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right)\beta_x^b(x, y) + \frac{5}{4}\left(z - \frac{4}{3h^2}z^3\right)\beta_x^s(x, y) \quad (8)$$

$$v(x, y, z) = v_0(x, y) + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right)\beta_y^b(x, y) + \frac{5}{4}\left(z - \frac{4}{3h^2}z^3\right)\beta_y^s(x, y) \quad (9)$$

$$w(x, y, z) = w_0(x, y) \quad (10)$$

The above unknowns are four rotations due to the bending and shear effects and three axial and transverse displacements. With the small strain assumptions, the displacement - strain relations can be achieved as

$$\varepsilon_x = u_{0,x} + z\frac{1}{4}(5\beta_{x,x}^s + \beta_{x,x}^b) + z^3\left(\frac{-5}{3h^2}\right)(\beta_{x,x}^s + \beta_{x,x}^b) \quad (11)$$

$$\varepsilon_y = v_{0,y} + z\frac{1}{4}(5\beta_{y,y}^s + \beta_{y,y}^b) + z^3\left(\frac{-5}{3h^2}\right)(\beta_{y,y}^s + \beta_{y,y}^b) \quad (12)$$

$$\varepsilon_{xy} = u_{0,y} + v_{0,x} + z\frac{1}{4}(5\beta_{x,y}^s + 5\beta_{y,x}^s + \beta_{x,y}^b + \beta_{y,x}^b) + z^3\left(\frac{-5}{3h^2}\right)(\beta_{x,y}^s + \beta_{y,x}^s + \beta_{x,y}^b + \beta_{y,x}^b) \quad (13)$$

$$\gamma_{yz} = \left(\frac{5}{4}\beta_y^s + \frac{1}{4}\beta_y^b + w_{,y}\right) + z^2\left(\frac{-5}{h^2}\right)(\beta_y^s + \beta_y^b) \quad (14)$$

$$\gamma_{xz} = \left(\frac{5}{4}\beta_x^s + \frac{1}{4}\beta_x^b + w_{,x}\right) + z^2\left(\frac{-5}{h^2}\right)(\beta_x^s + \beta_x^b) \quad (15)$$

or in a matrix form as

$$\begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\gamma} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\varepsilon}^{(0)} \\ \boldsymbol{\gamma}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \boldsymbol{\varepsilon}^{(1)} \\ \mathbf{0} \end{Bmatrix} + z^2 \begin{Bmatrix} \mathbf{0} \\ \boldsymbol{\gamma}^{(2)} \end{Bmatrix} + z^3 \begin{Bmatrix} \boldsymbol{\varepsilon}^{(3)} \\ \mathbf{0} \end{Bmatrix} \quad (16)$$

with the membrane strains are obtained from

$$\boldsymbol{\varepsilon}^{(0)} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} \quad (17)$$

The bending strains are given as

$$\boldsymbol{\varepsilon}^{(1)} = \frac{1}{4} \begin{Bmatrix} (5\beta_{x,x}^s + \beta_{x,x}^b) \\ (5\beta_{y,y}^s + \beta_{y,y}^b) \\ (5\beta_{x,y}^s + 5\beta_{y,x}^s + \beta_{x,y}^b + \beta_{y,x}^b) \end{Bmatrix} \quad (18)$$

$$\boldsymbol{\varepsilon}^{(3)} = \frac{-5}{3h^2} \begin{Bmatrix} \beta_{x,x}^s + \beta_{x,x}^b \\ \beta_{y,y}^s + \beta_{y,y}^b \\ \beta_{x,y}^s + \beta_{y,x}^s + \beta_{x,y}^b + \beta_{y,x}^b \end{Bmatrix} \quad (19)$$

The strains are then described as

$$\boldsymbol{\varepsilon}^{(0)} = \sum_{i=1}^4 \mathbf{B}_{1i} \mathbf{q}_i \quad (20)$$

$$\boldsymbol{\varepsilon}^{(1)} = \sum_{i=1}^4 \mathbf{B}_{2i} \mathbf{q}_i \quad (21)$$

$$\boldsymbol{\varepsilon}^{(3)} = \sum_{i=1}^4 \mathbf{B}_{3i} \mathbf{q}_i \quad (22)$$

$$\boldsymbol{\gamma}^{(0)} = \sum_{i=1}^4 \mathbf{B}_{4i} \mathbf{q}_i \quad (23)$$

$$\boldsymbol{\gamma}^{(2)} = \sum_{i=1}^4 \mathbf{B}_{3i} \mathbf{q}_i \tag{24}$$

in which

$$\mathbf{B}_{1i} = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$\mathbf{B}_{2i} = \frac{1}{4} \begin{bmatrix} 0 & 0 & 0 & 5N_{i,x} & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & 5N_{i,y} & 0 & N_{i,y} \\ 0 & 0 & 0 & 5N_{i,y} & 5N_{i,x} & N_{i,y} & N_{i,x} \end{bmatrix} \tag{26}$$

$$\mathbf{B}_{3i} = -\frac{5}{3h^2} \begin{bmatrix} 0 & 0 & 0 & N_{i,x} & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} & N_{i,y} & N_{i,x} \end{bmatrix} \tag{27}$$

$$\mathbf{B}_{4i} = \begin{bmatrix} 0 & 0 & N_{i,y} & 0 & \frac{5}{4} & 0 & \frac{1}{4} \\ 0 & 0 & N_{i,x} & \frac{5}{4} & 0 & \frac{1}{4} & 0 \end{bmatrix} \tag{28}$$

$$\mathbf{B}_{5i} = -\frac{5}{h^2} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{29}$$

where  $N_i$ ,  $N_{i,x}$  and  $N_{i,y}$  are the shape function and two derivatives of it in two directions  $x$ ,  $y$ . The element stiffness matrix

$$\mathbf{K}_e = \int_{\Omega_e} (\mathbf{B}_i^T \mathbf{D}^* \mathbf{B}_j + \mathbf{S}_i^T \mathbf{D}_s^* \mathbf{S}_j) d\Omega \tag{30}$$

with

$$\mathbf{B}_i = [(\mathbf{B}_{1i})^T \quad (\mathbf{B}_{2i})^T \quad (\mathbf{B}_{3i})^T] \tag{31}$$

$$\mathbf{S}_i = [(\mathbf{B}_{4i})^T \quad (\mathbf{B}_{5i})^T] \tag{32}$$

On the other hand, the element mass matrix is given as

$$\mathbf{M}_e = \int_{V_e} \mathbf{N}^T \mathbf{L}^T \rho(z) \mathbf{L} N dV = \int_{S_e} \mathbf{N}^T \left( \int_{-h/2}^{h/2} \rho(z) \mathbf{L}^T \mathbf{L} dz \right) N dS \tag{33}$$

with

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right) \frac{\partial}{\partial x} & \left(\frac{1}{4}z - \frac{5}{3h^2}z^3\right) \frac{\partial}{\partial y} & 1 \\ \frac{5}{4} \left(z - \frac{4}{3h^2}z^3\right) & 0 & 0 \\ 0 & \frac{5}{4} \left(z - \frac{4}{3h^2}z^3\right) & 0 \end{bmatrix}^T \tag{34}$$

Note that the element stiffness and mass matrices must be transformed from the local coordinate system to the global coordinate system as common sense. For vibration analysis, the equation can be described as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{35}$$

$$[\mathbf{K} - \omega^2 \mathbf{M}]\mathbf{q} = \mathbf{0} \tag{36}$$

More clearly, the finite element system of equations can be solved as followings:

- Input data: geometric data and material properties.
- Calculating constitutive matrix.
- Loop over elements: calculating element stiffness matrix and element mass matrix.
- Assembling all parts in the global coordinate system
- Applying boundary conditions
- Solving an equation for vibration
- Display normalized natural frequencies.

### 3. RESULTS AND DISCUSSION

In this part, the values of natural frequencies for FG cylindrical shells are calculated. Two kinds of boundary conditions are fully simply supported and fully clamped. Firstly, an FG cylindrical shell as shown in Figure 2a with clamped boundary condition and the material properties I in Table 1 as well as geometry properties of  $a/t = 10$ ,  $a/R = 0.1$  and  $a/b = 1$  is studied. The normalized fundamental frequencies  $\omega^* = \omega a^2 \sqrt{\rho_m t / D_m^*}$  with  $D_m^* = E_m t^3 / 12(1 - \nu_m^2)$  obtained by using the present finite element analysis are compared with [9] as Table 2. From Table 2 and Figure 3, it can be found that the obtained results match very well with the solutions in [9].

Secondly, a simply supported FG cylindrical shell with  $a/t = 2$ ,  $a/R = 0.5$  and  $a/b = 1$  is considered. The material properties are changed from I to II. The first normalized fundamental frequency  $\omega_1^* = \omega_1 t \sqrt{\rho_c / E_c}$  related to the proposed element is calculated and presented in Table 3. From Figure 4, it can be seen that in all cases, a close agreement exists between the results of the presented approach and the solutions in [10, 11]. The first three mode shapes for a quarter of this structure are illustrated in Figure 5.

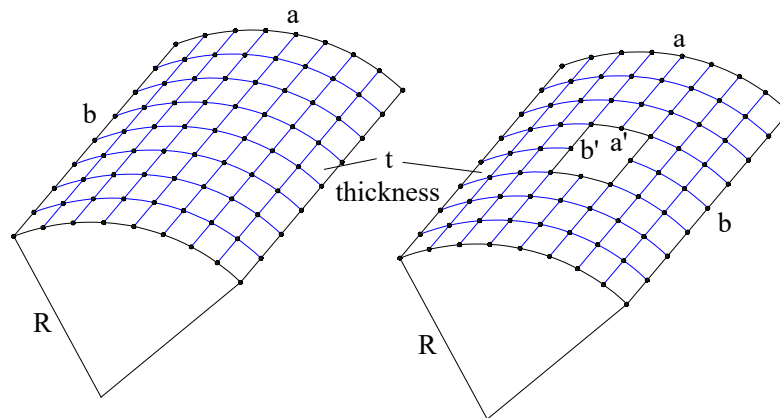


Figure 2. The  $8 \times 8$  mesh of functionally graded cylindrical shell (a) without and (b) with a central cutout

Table 1. The material properties

	Material I ( $\text{Si}_3\text{N}_4/\text{SUS304}$ )	Material II ( $\text{Al}_2\text{O}_3/\text{Al}$ )
$E_c$ (Pa)	$322.2715 \times 10^9$	$380 \times 10^9$
$\nu_c$	0.24	0.3
$\rho_c$ ( $\text{kg/m}^3$ )	2370	3800
$E_m$ (Pa)	$207.7877 \times 10^9$	$70 \times 10^9$
$\nu_m$	0.31776	0.3
$\rho_m$ ( $\text{kg/m}^3$ )	8166	2702

Table 2. The normalized fundamental frequencies of clamped  $\text{Si}_3\text{N}_4/\text{SUS304}$  cylindrical shell

Mode		$n$			
		0	0.2	2	10
I	Yang et al.	74.518	57.479	40.750	35.852
	Present	74.375	58.135	40.606	35.721
II	Yang et al.	144.663	111.717	78.817	69.075
	Present	144.479	111.649	78.904	69.113
III	Yang et al.	145.740	112.531	79.407	69.609
	Present	145.688	112.479	79.476	69.661

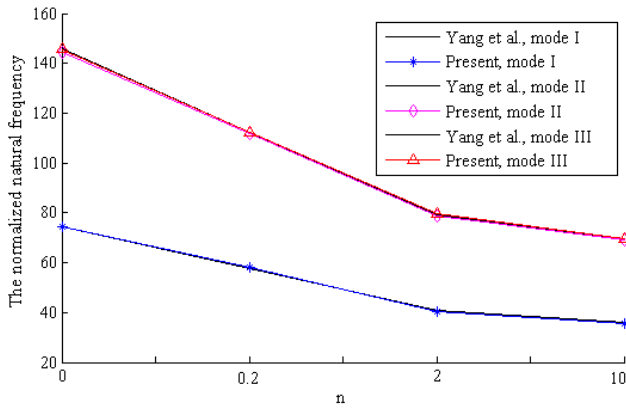


Figure 3. The normalized natural frequencies of a fully clamped FG Si<sub>3</sub>N<sub>4</sub>/SUS304 cylindrical shell

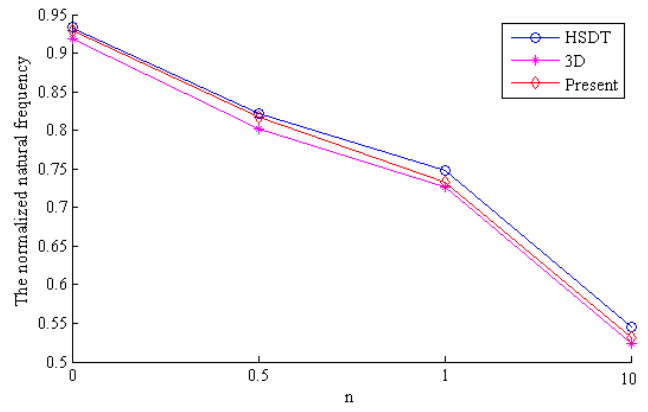


Figure 4. The normalized natural frequencies of a simply supported FG Al<sub>2</sub>O<sub>3</sub>/Al cylindrical shell

Table 3. The first normalized fundamental frequency of simply supported Al<sub>2</sub>O<sub>3</sub>/Al cylindrical shell

Method	<i>n</i>			
	0	0.5	1	10
HSDT	0.9334	0.8213	0.7483	0.5460
3D	0.9187	0.8013	0.7260	0.5245
Present	0.9294	0.8167	0.7329	0.5309

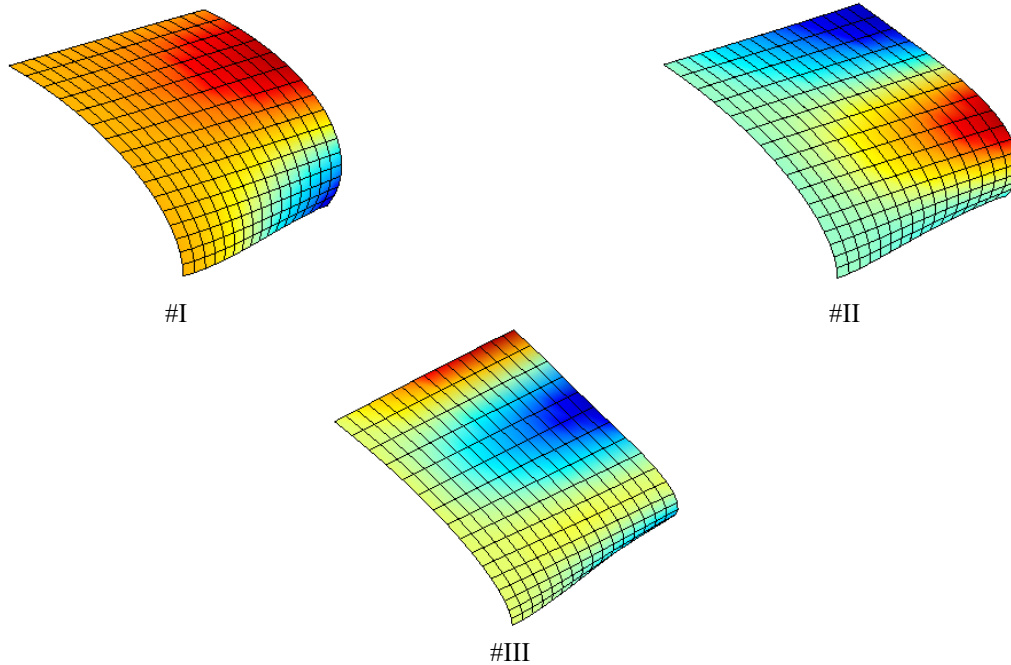


Figure 5. The first three mode shapes for a quarter of a simply supported FG Al<sub>2</sub>O<sub>3</sub>/Al cylindrical shell

Table 4. The first normalized fundamental frequency of clamped Si<sub>3</sub>N<sub>4</sub>/SUS304 cylindrical shell with a central cutout

Ratio of cutout <i>r</i>	Sai-Ram et al.	Present
0.1	33.886	34.109
0.3	38.330	39.004
0.5	57.689	58.072

Finally, another clamped FG cylindrical shell with central cutout as shown in Figure 2b is presented. This structure takes the material properties I with  $n = 0$  from Table 1 and with geometry properties of  $R/a = 5$  and  $a/b = 1$ . By changing the ratio of cutout  $r = a'/a = b'/b$ , the non-dimensional natural frequency  $\omega_1^* = \omega_1 a^2 \sqrt{\rho_c t / D_c^*}$  for mode I with  $D_c^* = E_c t^3 / 12(1 - \nu_c^2)$  is shown in Table 4 and then compared with [12]. Similarly, the presented results are in excellent agreement with others.

#### 4. CONCLUSION

In this paper, a novel quadrilateral element with four nodes and seven degrees of freedom per node in the framework of  $C^0$ -TSDT related to Shi's theory is presented to calculate the natural frequencies of FG cylindrical shells. Only the first derivative of transverse displacements is required. The results match well with the solutions from other published work. As the FG material has various benefits which leads to strong development, this article would provide essential knowledge for researchers in this area.

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