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Prediction of Bearing Service Life Using an Auto Regression Moving Average and Response Surface Methodology

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Abstract: Accurate prediction of service life of machines and their components is very important for reliability evaluation, and efficiency. It is intended in the current work to introduce a method to predict service life of bearings using accelerated life test rig. This is based on the measurement of vibration signals during an accelerated life test and applying an Auto Regressive Moving Average (ARMA) technique together with the Response Surface Methodology (RSM). Vibration signals are measured using accelerometers attached to deep groove ball bearings supporting a rotating shaft. Recorded signals are fed offline to a time-series routine within the SPSS software package where the coefficients of the ARMA models are estimated. Such models are usually utilized to predict the level of vibration signals, which is directly related to service life. The developed models are adequate enough to provide reasonable predictability measures. It is generally found that for all sets of data, the current response value is influenced by the past history of the preceding impacts and the residuals moving average. For better and comprehensive indication of the real functional interrelations between service lifetime of the tested bearing and the measured vibration signals, experimental signals amplitudes are represented in 3D surfaces and contour maps. It is found that signals amplitudes are drastically magnified near the end of bearing service life.

Keywords: Auto regression moving average (ARMA); Contour mapping; Reliability evaluation; Response surface methodology (RSM); Service life.

1. INTRODUCTION

Failures occurring during production processes result in negative implications. To overcome these implications; several methods to detect machinery faults have evolved to ensure equipment safety [1]. Methods for predicting machine element failures are divided into traditional reliability and prognostics approaches. The technical approaches to building models in prognostics are categorized broadly into data-driven approaches, model-based approaches, and hybrid approaches [2, 3, 4, 5].

Among the data-driven approaches is the Auto-regressive Moving Average (ARMA) models which is one of the most popular and frequently used stochastic time series models. The basic assumption made to implement this model is that the considered time series is linear and follows a particular known statistical distribution, such as the normal distribution. ARMA is widely reviewed as well as its applicability and simplicity of service life (SL) prediction of different machine elements [6, 7, 8]. ARMA gives a very broad and flexible family of stationary stochastic processes useful in representing many time series [9]. Among the advantages of ARMA models is the availability of the advanced ARMA related techniques for non- stationary data, also it is not required to understand detailed failure mechanisms plus it provides reliable predictions of service life [10, 11].

2. THE TECHNIQUE

ARMA is considered when hazard rate is a linear relationship of covariates and noise, short-term predictions are required, hazard rate is independent of age (i.e. exponential distribution), and measurement data is available for modelling and application but historical failure data is not [10].

Autoregressive models are developed in three recursive steps [10]:

a) Model identification: Using a set of time series data, values for the orders of the autoregressive and moving average parts of the ARMA equations are hypothesized, as well as the regular-difference parts for the ARMA model. A suitable criterion of fit is also declared.

b) Parameter estimation: Using non-linear optimization techniques (e.g., least- squares method), parameters of the ARMA equations are calculated to minimize the overall error between the model output and observed input-output data.
c) Model validation: A number of standard diagnostic checks are used to verify the adequacy of ARMA model, utilizing unseen data.

For the conditional expectations of the model, these three steps are repeated until a satisfactory model is obtained, which can then be used to forecast future values. Typical ARMA models (and variants) are effective for short-term predictions, but less reliable when used for long-term predictions [12]. The latter problem can be minimized by not using past predictions as the basis for estimating future predictions, which instead are entirely reliant on observations (i.e. condition monitoring or process data) [12, 13].

An ARMA (p, q) model is a combination of AR (p) and MA (q) models and it is suitable for univariate time series modeling. In an AR model the future value of a variable is assumed to be a linear combination of p past observations and a random error together with a constant term. Mathematically the AR model can be expressed as [8, 14]:

$$y_t = c + \sum_{i=1}^{p} (\phi_i\, y_{t-i}) + \varepsilon_t = c + \left(\phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p}\right) + \varepsilon_t \quad (1)$$

where y_t and ε_t are respectively the actual value and random error (or random shock) at time period t, $\phi_i (i = 1,2, p)$ are model parameters and c is a constant. The integer constant p is known as the order of the model. Sometimes the constant term is omitted for simplicity.

Just as an AR model regress against past values of the series, an MA model uses past errors as the explanatory variables. The MA model is given by [7, 8, 14]:

$$y_t = \mu + \sum_{j=1}^{q} (\theta_i\, \varepsilon_{t-j}) + \varepsilon_t = c + \left(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}\right) + \varepsilon_t \quad (2)$$

Here μ is the mean of the series, $\theta_j (j = 1,2, q)$ are the model parameters and q is the order of the model. The random shocks are assumed to be a white noise [7, 8] process, i.e. a sequence of independent and identically distributed random variables with zero mean and a constant variance.

Generally, the random shocks are assumed to follow the typical normal distribution. Thus, conceptually a moving average model is a linear regression of the current observation of the time series against the random shocks of one or more prior observations. Fitting a MA model to a time series is more complicated than fitting an AR model because in the former one the random error terms are not foreseeable [15]. Autoregressive (AR) and moving average (MA) models can be effectively combined together to form a general and useful class of time series models, known as the ARMA models [15]. Mathematically an ARMA (p, q) model is represented as [7, 8, 14]:

$$y_t = c + \varepsilon_t + \sum_{i=1}^{p} (\phi_i\, y_{t-i}) + \sum_{j=1}^{q} (\theta_i\, \varepsilon_{t-j}) \quad (3)$$

Here the model orders p and q refer to autoregressive and moving average terms respectively.

ARMA models play a key role in the modeling of time series. The linear structure of ARMA processes also leads to a substantial simplification of linear prediction. Compared with the pure AR or MA models, ARMA models provide the most effective linear model of stationary time series since they are capable of modeling the unknown process with the minimum number of parameters [16].

3. EXPERIMENTAL SETUP

An experimental setup as shown in Figure 1 has been designed and built to carry out an accelerated life test of deep groove ball bearings. Two ball bearings were used to support a 25 mm diameter steel shaft. The shaft was driven by a variable speed induction motor (380V/50Hz/1.5HP/2-poles/3500 rpm) via a flexible coupling. The free end of the shaft was loaded through a lever arm pivoted on another ball bearing. The arm was used to magnify the load transmitted to the free end of the shaft where an external load was applied on the free end of the arm. Figure 2 shows a schematic diagram of the test rig. Vibration signals were recorded in the vertical and horizontal directions for each of the two bearings *R* and *F*. The deep groove ball bearings were manufactured by FSB [12]. All vibration data readings were recorded at appropriate intervals of time using B & K accelerometers (Type 4381) having a voltage sensitivity equal to 8 mV/ms^{-2} and a frequency range up to 4800 Hz. Data collection was performed using a Brüel and Kjaer 4 Channel Pulse Analyzer (Type 2825) and Pulse Lap shop software (Version 6.1). Figure 3 shows a block diagram of the methodology followed in the present work.

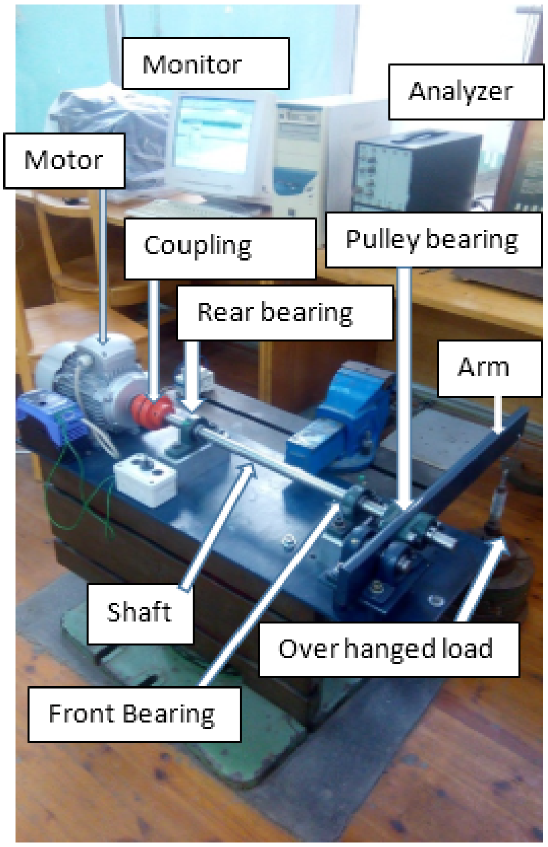


Figure 1. Image of the experimental setup

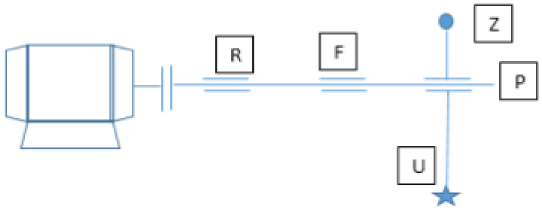


Figure 2. Schematic diagram of the test rig

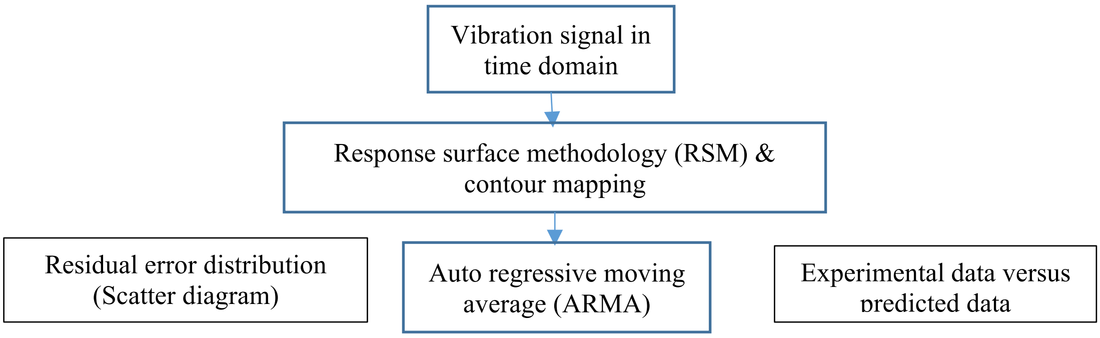


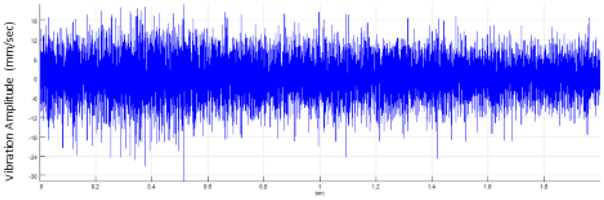
Figure. 3. Block diagram for the bearing performance and life prediction based on vibration signal using ARMA and RSM

4. RESULT, ANALYSIS, DISCUSSION AND EVALUATION

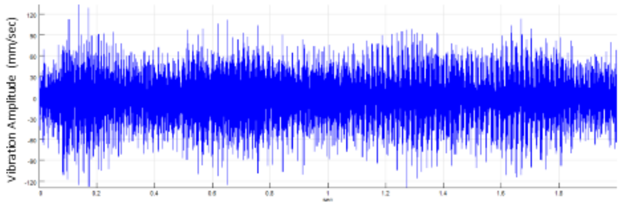
4.1 Time Domain Data Representation

Seven data samples at escalated service life intervals, $T = 0, 1, 25, 27.5, 33, 34.5$ and 36 hrs. at each of the front and the rear bearings positions are recorded. Each sampling period was two seconds long with a sampling rate of 4096 sample/sec resulting in 8192 data points in the time domain. For each location, signals are recorded in both horizontal and vertical directions to yield simultaneous four set of data group: front bearing at horizontal direction (FH), front bearing at vertical direction (FV), rear bearing at horizontal direction (RH), rear bearing at vertical direction (RV). A typical example of the time-domain signals for FH (first $T = 0$ and last $T = 36$ hrs. of bearing service life) are shown in Figure 4.

The amplitude of the signal at the end of the bearing life span was about much higher than those for the new bearing. This indicates a drastic degradation of the bearing dynamic performance. Although similar outcome is obtained for the recorded signals (FV, RH, and RV), the time-domain generally reflect only a qualitative indicator of the bearing health condition with low possible predictability.



(a) Typical sample from the recorded vibration signal FH after 1 hour



(b) Typical sample from the recorded vibration signal FH after 36 hours

Figure 4. Time-domain signals for the front horizontal location (FH) for first and last lifetimes samples

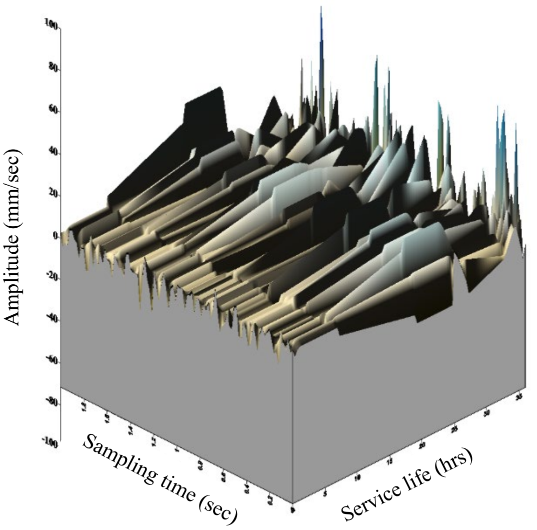
4.2 Response Surface Data Representation

According to vibration severity as per ISO 10816-1 class I for small machines, the unacceptable level range is 4.5-45.9 mm/sec RMS. In the present study, the life of the bearing is considered to end when the vibration level of the vibration signal reaches 22 mm/sec as shown in Table 1. To get a global view for data attitude along the whole life span of the bearing, the Response surface methodology (RSM) in terms of 3D surfaces and contour mapping representations are used.

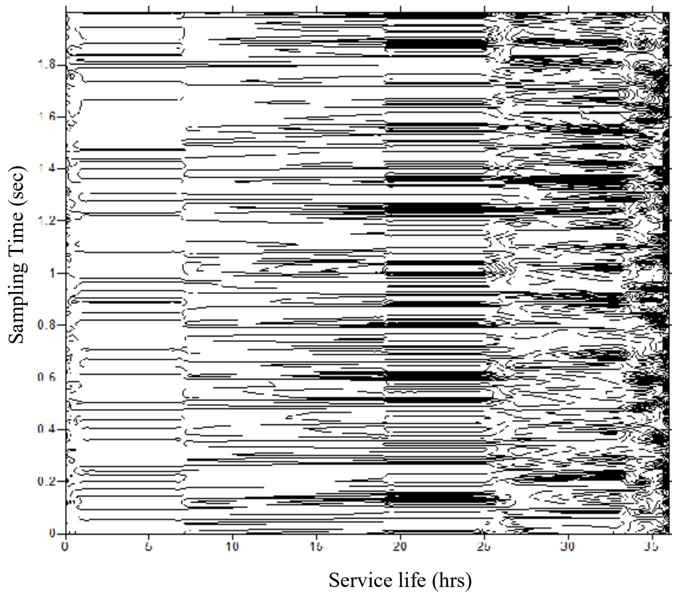
Figure 5a shows a 3D surface of the data, where the x-axis represents the sampling time which was 2 seconds. The y-axis represents the service life which is reached 36 hours in this case and the z-axis represents the vibration amplitude in mm/sec. The amplitude of the signal at 5% of the bearing life span was low within 20 mm/sec and by time the amplitude increased at 60% of the life span to be 60 mm/sec and reached the highest amplitude at the end of the bearing life span with 100 mm/sec. It is observed that the value of the amplitude was low at the beginning then it started to increase till it reached five times its value at a service life equal to 36 hours. This proves that the 3D surfaces representation of the vibration data reflects the bearing health condition.

Figure 5b shows the contour mapping of the vibration data recorded for the front bearing in the horizontal direction. The x-axis is the service life of the bearing measured in hours and the y-axis is the sampling time measured in seconds. The lines of the contour at 5% of the bearing life span are somehow far apart from each other. As the operation time 60% of the bearing service life, these lines are increased in number and became closer to each other till it reached the highest density at 100% of the bearing life span. On the other hand, levels seem hardly affected throughout the short sampling time of two seconds. This justifies how RSM may provide the designer with a better understanding of the attitude of time series data of non-stationary nature.

Similar response surface representations for the data for FV and those for both rear bearings RH and RV are shown in Figures 6-8. When comparing Figures 5 and 7 with Figures 6 and 8, it appears that the system is more sensitive to vibration in horizontal direction rather than in vertical direction and the system deterioration is greater in the horizontal direction. Moreover, the RMS values for FH shown in Table 1 marks the end of the experiment since it failed first. Service life of both front and rear bearings end simultaneously because of the mutual effect although they were subjected to relatively different loading conditions.



(a) 3D Surfaces of the data for FH at speed = 3500 rpm, and over hanged load = 305 N

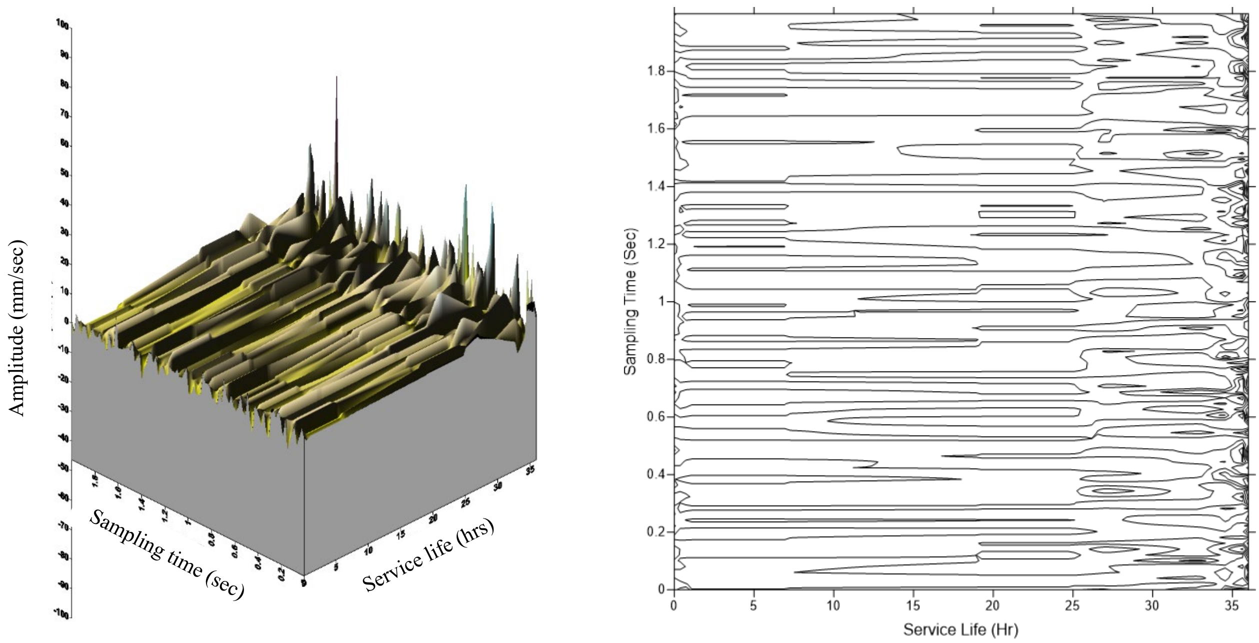


(b) Contour mapping of the Data for FH at speed = 3500 rpm, and over hanged load = 305 N

Figure 5. Response surface representation of the data for FH

Table 1. The overall vibration level in terms of RMS values (in mm/sec) along the whole service life of bearings at over hanged load = 305 N and speed = 3500 rpm

Hours	FH	FV	RH	RV
0	6.6	3.1	4.8	3.0
1	3.6	3.1	4.4	3.2
25	10.0	5.3	7.4	5.4
26	11.5	6.1	10.5	6.5
27	12.7	6.2	12.2	7.0
27.5	12.6	6.8	12.9	7.8
28	11.4	5.3	14.8	7.9
32	16.2	7.2	14.8	9.1
33	17.2	8.4	15.8	9.4
33.5	18.6	8.1	14.4	9.5
34	19.3	7.6	16.7	9.4
34.5	20.8	9.1	18.6	10.2
35	21.8	10.1	19.5	10.7
36	22	11.7	21.4	11.3



(a) 3D surfaces for FV at speed = 3500 rpm, and over hanged load = 305 N

(b) Contour mapping for FV at speed = 3500 rpm, and over hanged load = 305 N

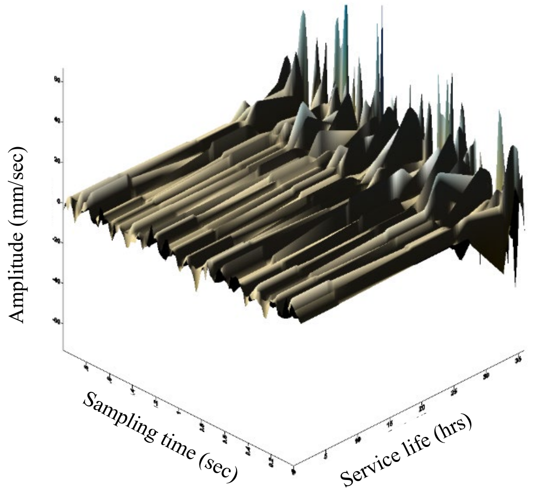
Figure 6. Response surface representation of the data for FV

4.3 Autoregressive Moving-Average (ARMA) Data Modelling

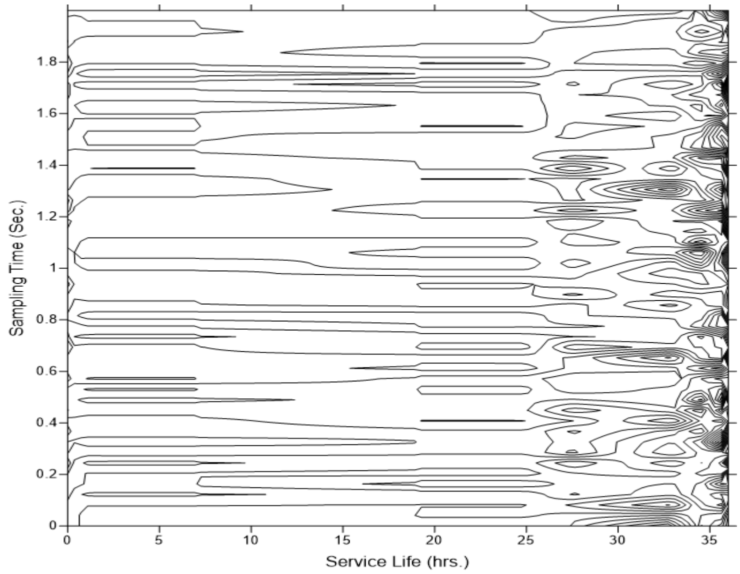
ARMA modelling is reported as an efficient robust diagnostic tool for experimental data of systems with dynamic nature [17]. Signals are fed offline to the time-series routine within the SPSS computing program package and, the ARMA models' coefficients are determined. The developed ARMA model is in the form:

$$X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + \cdots + \phi_n X_{t-n} = \mu + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \cdots - \theta_m a_{t-m} \quad (4)$$

where δ and μ are the constant coefficients for AR and MA respectively, ϕ_i and θ_j are coefficients for AR and MA respectively and a_t is the residual zero mean.

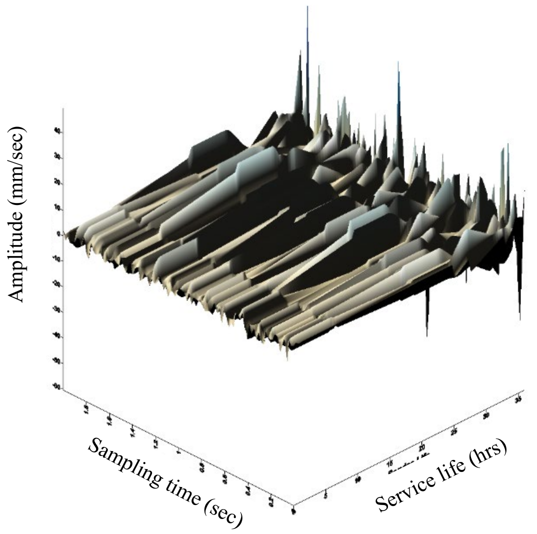


(a) 3D surfaces for RH at speed = 3500 rpm, and over hanged load = 305 N

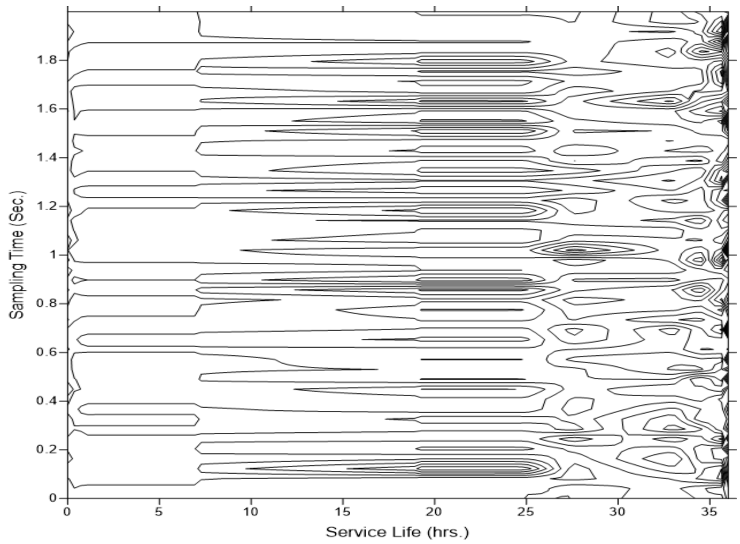


(b) Contour mapping for RH at speed = 3500 rpm, and over hanged load = 305 N

Figure 7. Response surface representation of the data for RH



(a) 3D surfaces for RV at speed = 3500 rpm, and over hanged load = 305 N



(b) Contour Mapping for RV at speed = 3500 rpm, and over hanged load = 305 N

Figure 8. Response surface representation of the data for RV

Table 2 lists a summary of the model outcome after extensive individual data samples at various service lifetime for FH signals. The table shows also shows the values of the determination (R^2) along the lifetime of the bearings operated under accelerated life test as well as the values of the coefficients besides the value of the overall ARMA of the model. ARMA model indicates to what extent the current response value X_t is influenced by the past history of the preceding impacts. Different models resulted for various service lifetime intervals. For example, for data captured at 25 hours ($T = 25$, Table 1), ARMA (3,8) results indicating that the estimation of the instantaneous vibration level is associated back with the dynamic impacts of the preceding three impacts AR(3) with the residuals from the mean of the preceding eight impacts MA(8). However, at the end of service lifetime $T = 36$, an ARMA (6,8) results indicating the prediction dependency on farther preceding impacts AR(6). Additional model is postulated considering the entire recorded data, $T = 0$ to 36 including 57344 data points, with a resulting ARMA (5,7) overall model.

Model adequacy and significance are usually judged through their predictability, the determination factors of the developed model and the residuals' behavior. As indicated in Table 2, the values of determination coefficient R^2 of the models are in the range from 0.611 to 0.804. These are considered reasonable measures for model goodness of fit considering the high amount of data points in association and the dynamic non-stationary nature of the signals. As indicated by the R^2 values shown in Table 2, nothing appears against the model adequacy and the residuals follow an approximate random normal distribution with zero mean and 15.05 standard deviation as shown in Figure 9. High model predictability is also examined through the comparison between the experimental (observed) data and their corresponding predicted values as represented by Figure 10. Good and sufficient match is rather obtained at advanced stages of bearing service life. Similar procedures are followed to obtain ARMA models for the rest of data FV, RH, and RV. The outcomes are summarized in Tables 3-5.

Table 2. Results of ARMA for the case of FH

	Model (T)	0	1	25	28	33	35	36	Overall Model
	ARMA	(0,16)	(5,8)	(3,8)	(8,8)	(4,7)	(0,8)	(6,8)	(5,7)
	R^2	0.611	0.600	0.672	0.713	0.610	0.688	0.804	0.680
Coefficients	δ	0.313	0.105	0.297	0.106	0.079	0.120	0.186	0.172
	ϕ_2			-0.987					
	ϕ_3			-0.110					
	ϕ_4		0.090			-0.449			0.388
	ϕ_5		-0.560					0.134	0.337
	ϕ_6							-0.053	
	ϕ_7				0.064				
	ϕ_8				0.120				
	θ_1	-0.455	-0.358	-0.145	-0.679	-0.070	-0.105	-0.227	-0.205
	θ_2	1.000	0.944	-0.031	1.190	1.060	1.480	1.760	1.320
	θ_3	0.255	0.102	-0.441	0.529	-0.198		0.264	-0.015
	θ_4	0.175	-0.280	1.310	-0.434	-0.363	-0.249	-0.829	0.130
	θ_5	0.518	0.269	0.215	0.051	-0.074	0.435	0.243	0.777
	θ_6	-0.204	0.341		0.142	0.261	-0.180	0.101	-0.488
	θ_7	-0.298		0.385	0.093	0.353	-0.322	-0.269	-0.550
	θ_8		-0.074	-0.342	0.073		-0.083	-0.058	
	θ_{16}	0.019							

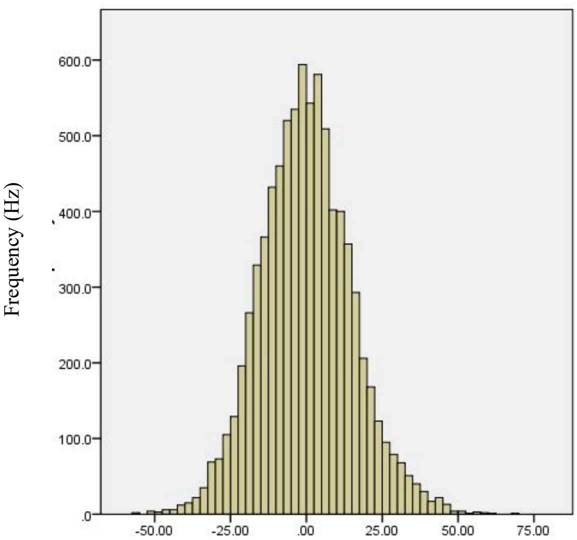


Figure 9. Noise residual for FH at $T = 36$ hrs. follow an approximate random normal distribution

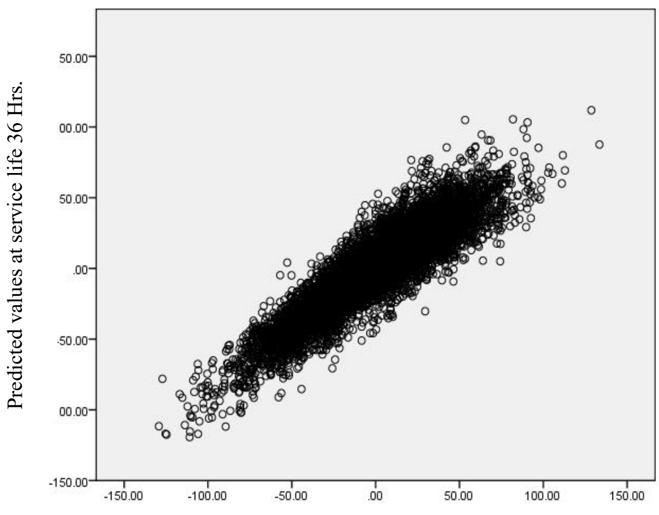


Figure 10. Model predictability

Table 3. Results of ARMA for the case of FV

	Model (T)	0	1	25	28	33	35	36	Overall Model
	ARMA	(0,8)	(4,7)	(0,8)	(3,7)	(3,8)	(4,8)	(6,8)	(3,8)
	R^2	0.677	0.570	0.624	0.640	0.760	0.723	0.633	0.568
Coefficients	δ	0.249	0.053	0.287	0.363	0.089	0.308	0.522	0.267
	ϕ_1								
	ϕ_2				-0.789	-0.366			-0.299
	ϕ_3		0.504		-0.054	0.542	0.280	-0.120	0.067
	ϕ_4		-0.359				0.071		
	ϕ_5								
	ϕ_6							-0.266	
	θ_1	-0.955	-0.584	-0.856	-0.775	-0.521	-0.633		-0.343
	θ_2	0.116	0.619	-0.599	-0.183	0.722	1.195	1.307	0.695
	θ_3	0.164	0.939	0.555	-0.473	0.436	0.878	-0.077	0.143
	θ_4	0.588	0.348	0.435	0.648	0.478	0.224	-0.102	0.480
	θ_5	0.733	-0.199	0.405	0.757	0.282	0.172		0.336
	θ_6	0.325			0.463		-0.271	-0.441	0.026
	θ_7	0.055	-0.160	-0.102	0.451	-0.180	-0.428	0.088	-0.194
	θ_8	-0.071		-0.064		-0.238	-0.179	0.189	-0.319

Table 4. Results of ARMA for the case of RH

	Model (T)	0	1	25	28	33	35	36	Overall Model
	ARMA	(0,10)	(2,8)	(3,8)	(2,10)	(6,8)	(2,8)	(2,7)	(2,8)
	R^2	0.715	0.716	0.722	0.751	0.794	0.862	0.715	0.670
Coefficients	δ	0.162	0.020	0.287	-0.078		0.045	-0.017	0.032
	ϕ_1		1.651		0.959		1.684	0.572	1.207
	ϕ_2		-0.919	-0.279	-0.999		-0.953	-0.109	-0.916
	ϕ_3			-0.616					
	ϕ_4					-0.078			
	ϕ_5					-0.345			
	θ_1	-0.990	-0.902	-0.979	0.252	-0.734	-0.893	0.060	-0.324
	θ_2	0.143	0.906	0.124	0.524	1.037	1.393	1.396	0.714
	θ_3	0.087	-1.165	-0.557	0.832	0.826	-1.506	-0.124	-1.177
	θ_4	0.397	0.193	-0.053	0.663	0.686	-0.249		0.298
	θ_5	0.732	0.050	0.805	0.614	0.623	0.508	0.256	0.536
	θ_6	0.303		0.354	0.116	-0.693	0.084	-0.423	-0.277
	θ_7	0.178	0.302	0.711		-0.731	0.131	-0.177	0.326
	θ_8	0.135	-0.196	0.547		-0.057	-0.257		0.193
	θ_{10}	-0.016			-0.358				

5. CONCLUSION

An accelerated life test was performed on deep groove ball bearings using previously designed test rig. Dynamic characteristics (vibration signals) for front and rear ball bearing of a rotating shaft were recorded. The use of RSM in term of 3D surfaces and contour mapping provides a global view for data attitude along the whole life spans of the bearing and gives an evaluation of the state of health of the bearings by indicating how signals amplitudes are drastically magnified near the end of bearing service life. The recorded signals were fed offline to the time-series routine within the SPSS computing program package and, the ARMA models' coefficients were determined. These ARMA coefficients can be used to predict the service life of a similar bearings subjected to the same working conditions. The values of coefficient of determination R^2 of the ARMA models ranging from 0.611 to 0.804 for the front bearing in the horizontal direction can be considered as reasonable measures for model's goodness of fit considering the high amount of data points in association and the dynamic non-stationary nature of the signals. It is generally found for all sets of the data that the current response value is influenced by the past history of the preceding impacts and the residuals moving average. The developed ARMA models can be considered as a reasonable predictability measure.

Table 5. Results of ARMA for the case of RV

	Model (T)	0	1	25	28	33	35	36	Overall Model
	ARMA	(0,7)	(0,10)	(0,10)	(3,6)	(1,5)	(6,8)	(2,7)	(2,7)
	R^2	0.580	0.657	0.643	0.681	0.627	0.788	0.704	0.670
Coefficients	δ	0.204	0.030	-0.063	0.309	0.135	0.287	0.284	0.170
	ϕ_1				1.867	0.636		1.439	1.486
	ϕ_2				-1.368			-0.667	-0.722
	ϕ_3				0.358				
	ϕ_5						-0.104		
	ϕ_6						-0.299		
	θ_1	-0.384	-0.511	-0.811	1.439	0.685	-0.684	0.974	0.880
	θ_2	1.145	1.190	0.636	0.605	0.988	1.206	1.219	1.233
	θ_3	0.221	0.476	0.441	-2.227	-0.389	1.140	-1.505	-1.405
	θ_4		-0.050	0.318	1.057		0.372		-0.103
	θ_5	0.357	0.154	0.464	0.844	-0.293	-0.140	0.634	0.531
	θ_6	-0.189	-0.224	0.144	-0.726		-0.652	-0.275	-0.047
	θ_7	-0.191	-0.127	-0.088			-0.322	-0.051	0.032
	θ_8			-0.213			0.040		-0.150
	θ_{10}		0.044	0.089					

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