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Fuzzy Logic Control of a Rotary Double Inverted Pendulum System

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Abstract: This work presents a fuzzy logic control of a rotary double inverted pendulum (RDIP) system. The RDIP system consists of two inverted pendulums mounted on a rotating disc which is driven by a DC motor. The longer pendulum is hinged on the right side of the disc whereas the shorter pendulum is hinged on the left side of the disc. The RDIP is an extremely nonlinear, unstable and under-actuated system of high order. A mathematical model is built for the RDIP using the Lagrange-Euler equation. A Fuzzy Logic Controller is then designed for swing-up and stabilization control of the RDIP system and its stability analysis is presented. The fuzzy controller takes the angles and angular velocities of the two pendulums and the angle and angular velocity of the rotary disc as its inputs, and the driving force as its output. A PID controller is also developed for this system for the purpose of comparison. The simulation results of these two control schemes with their comparative analysis show that, although both of the classical PID and the fuzzy controllers can control the system properly, the latter performs better especially on the steady state behavior. The simulation results also show the capability of the fuzzy logic control strategy to control a highly nonlinear and unstable system such as the RDIP.

Keywords: Fuzzy logic control; Rotary inverted pendulum; PID; Simulation.

1. INTRODUCTION

Inverted pendulum systems are often used as a benchmark for verifying the effectiveness of a new control method because of the simplicity of their structure and complex nonlinear dynamics. The families of inverted pendulum systems are classified on the basis of the number of pendulums and how they are linked. Thus there are single inverted pendulum [1-2], double inverted pendulum, triple inverted pendulum systems, etc. In particular in double inverted pendulum system, the two pendulums can be linked in series [3-5] or in parallel [6-7]. In this case they are called series-type double inverted pendulum and parallel-type double inverted pendulum. In the latter case the two linked pendulums are mounted on a moving cart or a rotating disc. Furthermore, in case of pendulum system with rotating disc the disc rotation affects the two pendulums directly and the shorter pendulum with a higher natural frequency tends to respond intensively and is liable to fall down. Therefore, it is said that stabilization control of a rotary parallel-type double inverted pendulum system is the most difficult to handle among the family of inverted pendulum systems [8].

RDIP is a Single-Input-Multi-Output system. Its dynamics is very nonlinear and unstable. Therefore, RDIP system is very hard to control, especially by conventional controller such as PID. It is expected that by using fuzzy logic controller, the RDIP system can be controlled with satisfactory performance. In this work, we propose the use of Fuzzy Logic Control to address the issue of swing-up and stability for RDIP system. All simulations were done using MATLAB.

2. SYSTEM MODELLING AND SIMULATION

2.1 System Description

As shown in Figure 1, a parallel-type double inverted pendulum system consists of a rotating disc which is driven by a DC motor, longer pendulum hinged on the left side of the disc and a shorter pendulum hinged on the right side of the disc. In the same vertical plane with the disc, the two pendulums can rotate freely around their own pivots.

Figure 1. RDIP physical system (www.googoltech.com)

Table 1. Parameters of rotary double inverted pendulum system

Parameters	Notation	Values	Units
Inertia of the rotating disc	J_0	0.06	kg-m^2
Inertia of the first pendulum	J_1	0.008	kg-m^2
Inertia of the second pendulum	J_2	0.002	kg-m^2
Viscous coef. of rotating disc	c_0	0.004	N-m-s
Vicious coef. of the first pendulum	c_1	0.0031	N-m-s
Viscous coef. of the second pendulum	c_2	0.00088	N-m-s
Mass of the first pendulum	m_1	0.25	kg
Mass of the second pendulum	m_2	0.13	kg
The displacement from the joint to the C.M. of the first pendulum	l_1	0.24	m
The displacement from the joint to the C.M. of the second pendulum	l_2	0.13	M
The radius of the rotating disc	L	0.172	m
The gravity constant	g	9.8	m/s
Torque constant	K_m	0.005	N.m/A
Back emf. constant	K_b	0.001	V/rad
Resistance in motor circuit	R	2	Ω

2.2 System Mathematical Model

Figure 2 shows vector representatives in Cartesian and vector coordinate for modeling of RDIP where C.M is the center of mass of pendulum, (e_x, e_y, e_z) is the set of unit vector of the Cartesian coordinate and (e_r, e_α, e_z) is the set of unit vectors in the angular coordinate. $e_{r'}$ is unit vector in the direction along the pendulum rod.

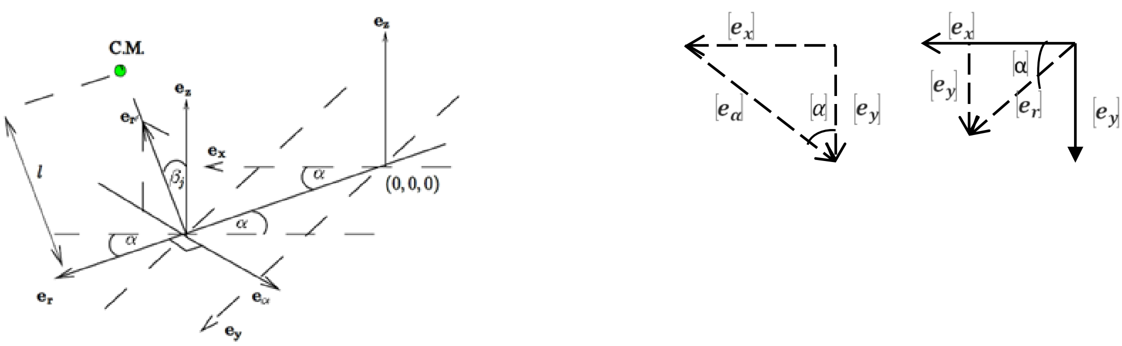


Figure 2. Vector representatives in Cartesian and vector coordinate

Mathematical model of the RDIP system is given by Equation (1) [8] where all parameters are given in Table 1.

$$E\dot{x} \; = Fx + Gu$$

$$\dot{x} \; = \; E^{-1}Fx + \; E^{-1}Gu =: Ax + Bu \tag{1}$$

Where,

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_0 + m_1L^2 + \; m_2L^2 & -m_1l_1L & -\,m_2l_2L \\ 0 & 0 & 0 & -m_1l_1L & J_1 + \; m_1l_1^2 & 0 \\ 0 & 0 & 0 & -m_2l_2L & 0 & J_2 + \; m_2l_2^2 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -(C_0 + \dfrac{K_mK_b}{R}) & 0 & 0 \\ 0 & m_1gl_1 & 0 & 0 & -C_1 & 0 \\ 0 & 0 & m_2gl_2 & 0 & 0 & -C_2 \end{bmatrix}$$

$$G = [0 \quad 0 \quad 0 \quad \frac{K_m}{R} \quad 0 \quad 0\,]^T$$

2.3 Open-Loop Model Verification

The mathematical model of the system used is given in Section 2.2 [8]. The simulation is done using Simulink in MATLAB environment. The ode45 solver in MATLAB is used here in all cases to solve the ordinary differential equations. The open-loop model of the RDIP is shown in Figure 3.

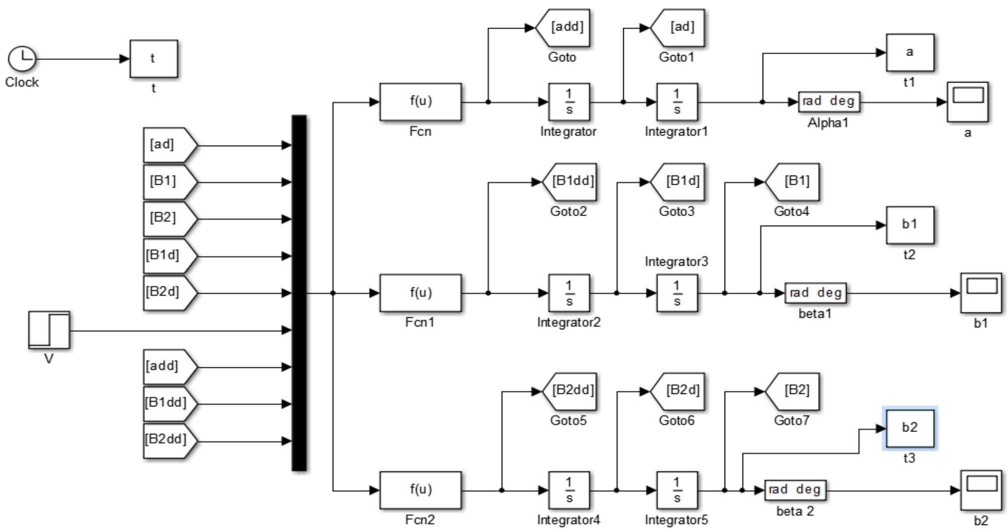
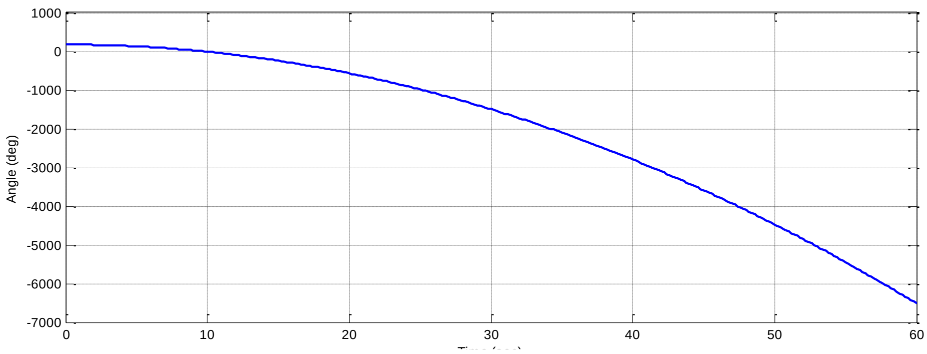


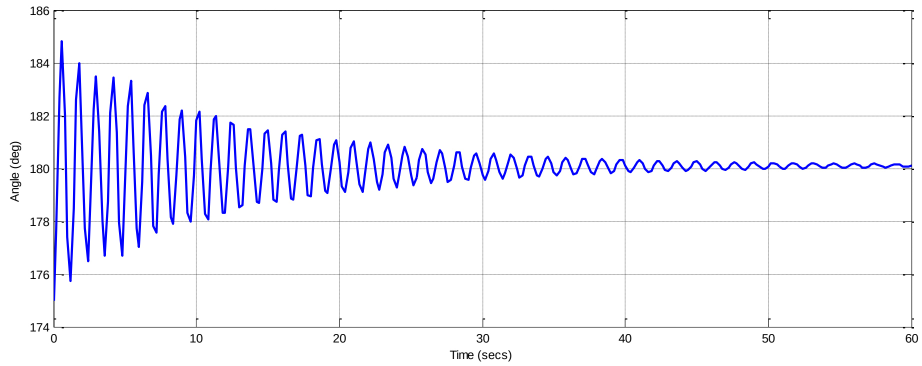
Figure 3. Open-loop RDIP Simulink model

The Simulink model in Figure 3 has one input (*v*) and three outputs (*a*, *b1*, *b2*), where *v* is the input voltage, *a* is the rotary disc angle as viewed in a scope, *b1* and *b2* are first and second pendulums angles viewed in scopes. The variables *ad* and *add* are respectively first and second derivatives of rotary disc angle. *B1*, *B1d*, *B1dd*, *B2*, *B2d*, *B2dd* are the first pendulum angle, first and second derivatives of the first pendulum angle, second pendulum angle, first and second derivatives of the second pendulum angle respectively.

The initial conditions for the two pendulums are 180 degrees which means the pendulums are in the downward position at the starting point. The states in all the figures will be shown in rad and rad/s for easy observation. The open-loop (without controller) response of the RDIP model is shown in Figure 4. The simulation time is from 0 s to 60 secs. As can be seen from Figure 4 when there is no control, the two pendulums fall down directly to their stable equilibrium where the RDIP is in its lowest total energy state ($\beta_1 = \beta_2 = 180^0$).



(a)



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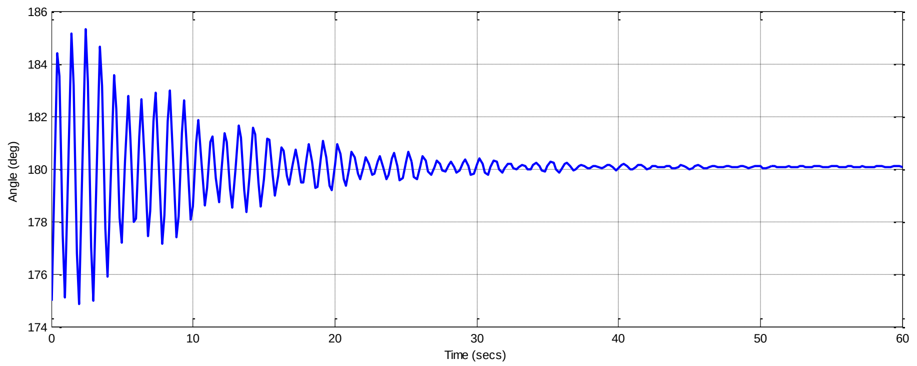


(c)

Figure 4. Open-loop response of the RDIP model: (a) Plot of α, (b) Plot of $\beta_{1,}$ (c) Plot of β_2

3. FUZZY LOGIC CONTROLLER DESIGN

In design of fuzzy logic controller, Mamdani method is used. Three fuzzy controllers are designed to control the two RDIP pendulums and the rotary disc, denoted by FL1, FL2, and FL, respectively as shown in Figure 5. The fuzzy logic controller FL1 inputs are error of first pendulum angle (β_1), error of first pendulum angular velocity ($\dot{\beta}_1$), fuzzy logic controller FL2 inputs are error of second pendulum angle (β_2), error of second pendulum angular velocity ($\dot{\beta}_2$), fuzzy logic controller FL inputs are rotating disk angle (α), and rotating disk angular velocity ($\dot{\alpha}$). The fuzzy logic controller output is control signal in Volt unit. Error value is gained from the deviation between angle and set point. The set point is when the pendulum in the upright position (0^0).

When output gain adjustment was inadequate, the membership functions were modified for fine-tuning. Thus 25 rules for each pendulum and 49 rules for rotary disc resulting from these modifications in membership functions and rule base gave the responses that best stabilize the pendulums. Centroid of area (COA) is the defuzzification process used in this work. Membership function plots for α is shown below in Figure 6. Similarly, membership functions for $\dot{\alpha}$ (DERROR), control signal (output), β_1 and β_2, $\dot{\beta}_1$and $\dot{\beta}_2$ and control signal (output1) were obtained. The 25 rules for the pendulums are summarised in Table 2, and the 49 rules for rotary disc are summarised in Table 3.

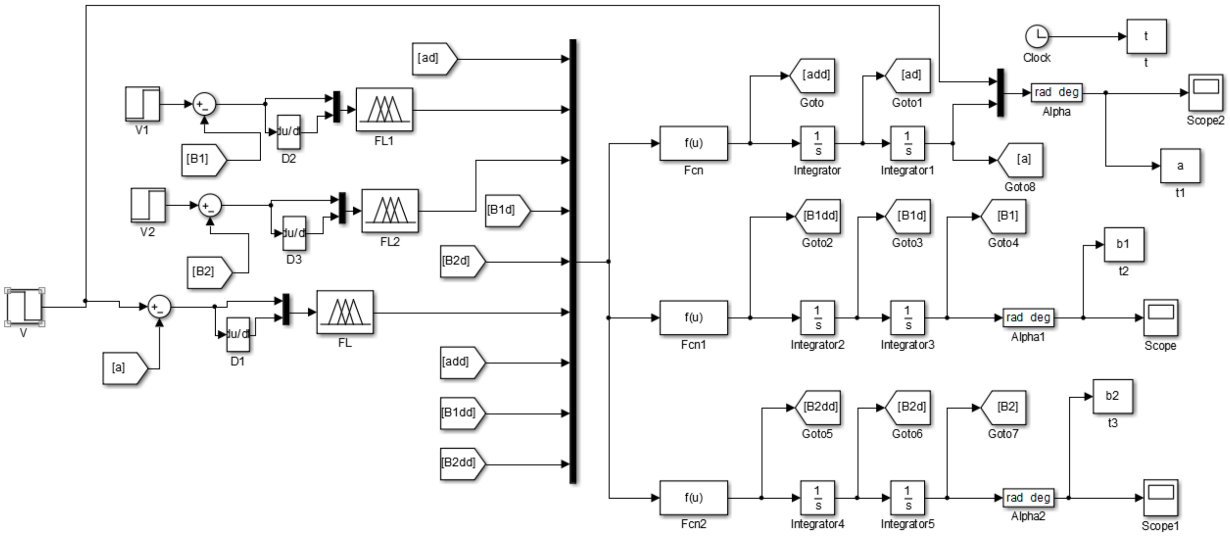


Figure 5. Fuzzy logic controller and RDIP

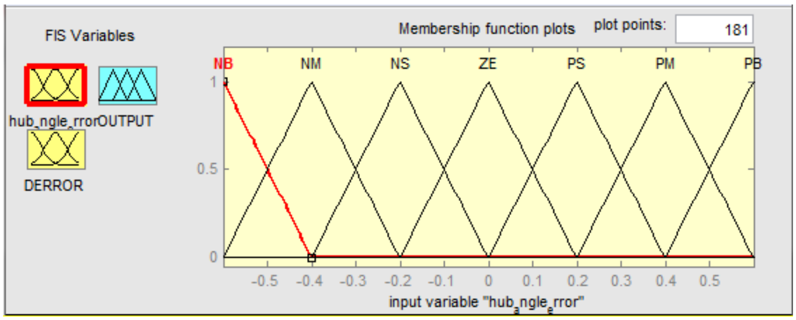


Figure 6. Membership function α (hub_angle_error)

Table 2. Rule base table for pendulums

$e\backslash\dot{e}$	NB	NS	ZE	PS	PB
NB	NB	NB	NB	NS	ZE
NS	NB	NB	NS	ZE	PS
ZE	NB	NS	ZE	PS	PB
PS	NS	ZE	PS	PB	PB
PB	ZE	PS	PB	PB	PB

Table 3. Rule base table for rotary disc

$e\backslash\dot{e}$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	ZE	PS
NS	NB	NB	NM	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB

3.1 PID Control

A PID controller for RDIP system was designed for a performance comparison and as according to [9]. The feedback control gain is given by Equation (2) using Linear Quadratic Regulator technique with the weighing matrices given in Equation (3) and it was successful in swing up and stabilization control of the system. The PID controller parameters were obtained as [9]:

$$k = [1.28, \quad -42.49, \quad -103.15, \quad 1.88, \quad -11.94, \quad -11.72, \quad 0.32] \tag{2}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 10 \tag{3}$$

For the rotary disc, a reference angle of 45^0 was set. The Simulink model for the PID control of the rotary disc angle and the pendulums angles are shown in Figure 7.

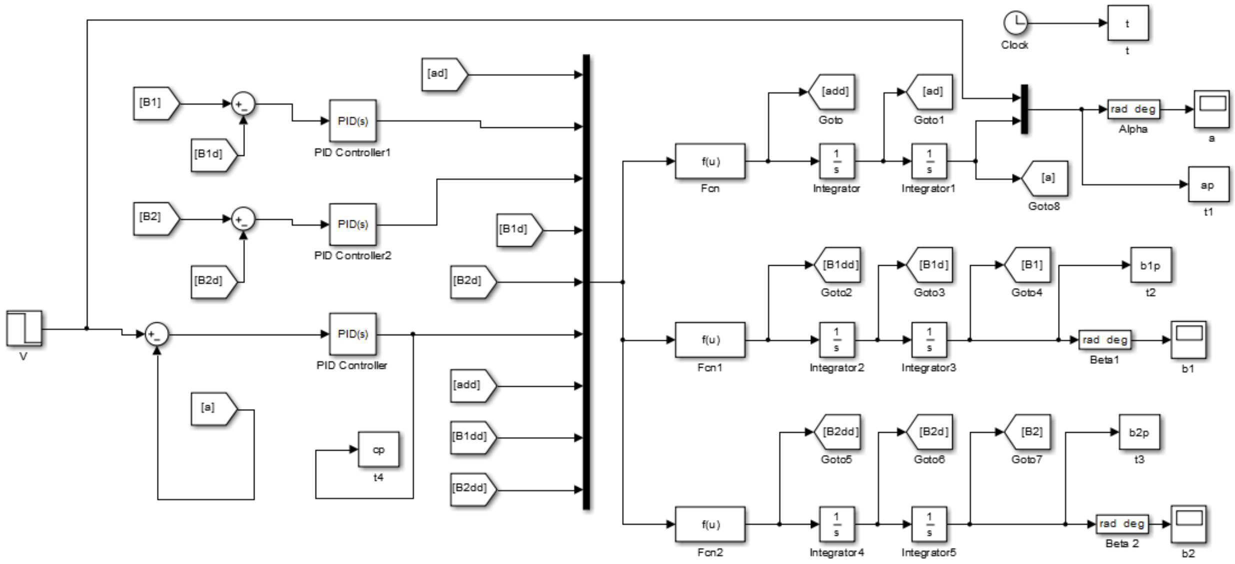


Figure 7. Simulink model of RDIP system subjected to PID control

4. RESULTS AND ANALYSIS

The FLC versus PID control signal is shown in Figure 8. It can be seen that the control signal of the PID method oscillates. It is found that there is a sudden change at the beginning of the control process, which leads to an overshoot. Since a high feedback gain may lead to torque saturation, noise amplification, and other problems in the experiment, there is a need to avoid it in future. In addition, the control signal of the PID has a magnitude much larger than the FLC. This means the FLC controller will save energy used to control the system. Figure 9 shows the merged responses of both PID and Fuzzy Controllers for rotary disc, longer pendulum and shorter pendulum respectively.

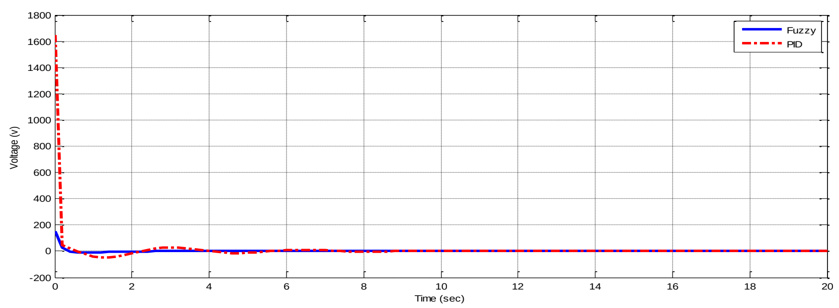
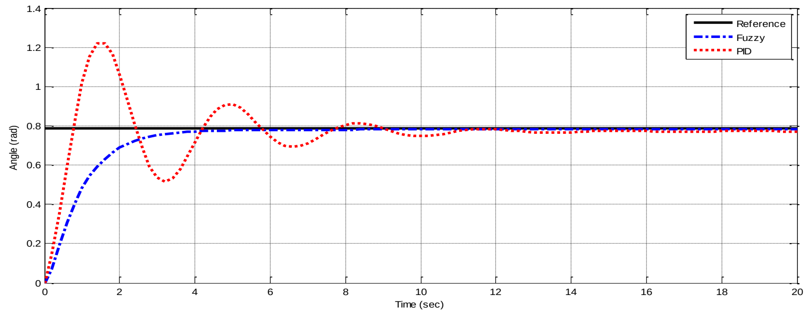
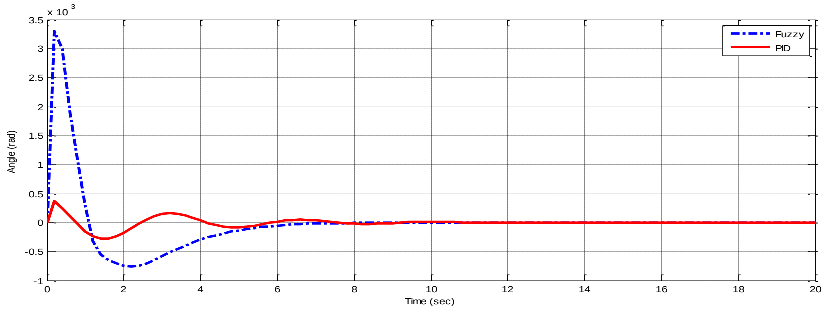


Figure 8. FLC and PID control signals



(a)



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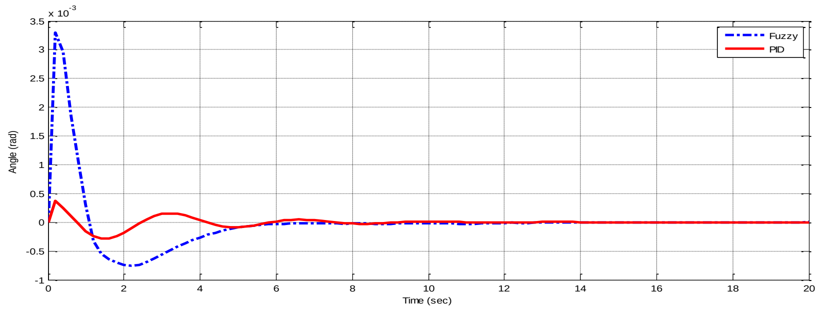

(c)

Figure 9. Responses of Fuzzy and PID controllers: (a) Rotary disc, (b) Longer pendulum, (c) Shorter pendulum

The simulation results are summarized in Table 4. It is noted that the FLC control generates no overshoot. In comparison, the PID control has 40% overshoot. In addition, the settling time due to FLC control is within 6 secs compared to that of PID which is 13 secs. However, the PID controller has less rise time of 1 sec compared to FLC controller which has a rise time of 4 secs.

Table 4. Summary for step response

Controller	Rise Time (sec)	Overshoot (%)	Settling Time (sec)	Steady State Error
FLC	4	0	6	0
PID	1	40	13	0

5. CONCLUSION

Two control approaches for the RDIP system have been designed and simulated to swing-up and balance a RDIP. It was noted that the classical PID and the fuzzy controllers can control the system properly, although the second controller performed better than the classical PID controller. Both controllers show almost a zero amount of steady state error. Simulation results show that the fuzzy controller completely stabilizes the parallel type double inverted pendulum system in 6 secs compared with the PID controller where stabilization of the system is after 13 secs. The fuzzy method investigated has some advantages with respect to the conventional PID method. It allows for more design flexibility as clearly illustrated by this work design. In summary, the FLC displays an improved behavior in comparison with the conventional PID technique. The FLC approach has also been proved to be successful in control of the nonlinear, unstable, and interactive double pendulum system.

The principle employed in this work can be used to swing-up and balance a triple pendulum system. As an extension to future work, effects of variation in type of membership functions on RDIP can be considered further. The FLC can also be improved in some other ways. Incorporation of heuristics about the inverse plant dynamics to speed the adaptation is an approach that can be applied. Inverse plant is a fuzzy system that is heuristically designed to approximate roughly a plant's inverse dynamics. The detail on this issue is discussed in [10]. It is still necessary to evaluate the performance of the FLC under a greater variety of conditions. Robustness of the controller against many different types of disturbances is yet to be investigated for instance, how the fuzzy controllers react to a white noise disturbance in the control input has not been studied in this work. One can implement these control schemes in experiment to verify the simulation results, which will be a great challenge since in the experiment; there may be some disturbance, unknown dynamics or other factors.

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