

# LMI-Based Control of a Double Pendulum Crane

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**Abstract:** This paper presents the design of a Linear Matrix Inequality (LMI) based state feedback controller for position tracking, hook and payload oscillations of a double pendulum crane. In this work, a linearised model of the crane was firstly obtained. The idea of formulating the stability condition in the form of LMIs using the linearised model was proposed. Using this approach, the stabilisation conditions constraints the closed-loop poles of the system to be in an LMI region to guarantee the system stability and gives satisfied transient performance. Based on the stabilization conditions formulated, an LMI based state feedback tracking controller for trolley position tracking and swing angles control of the double pendulum crane is proposed. Mostly, two or more controllers were used for the control of double pendulum crane to control the trolley, hook and payload. However, in this work a single LMI based controller is used to achieve similar performance. This reduces the number of controllers which minimises cost, complexity and size of the control system. The performance of the proposed controller is investigated via simulations. The simulation result shows the proposed controller is able to track the trolley position relatively fast with minimum hook and payload swing angles hence, reduce the main problem of a double pendulum crane. From the performance comparison of the results, the proposed method has improved in terms of reducing the trolley position percentage overshoot, hook oscillation and payload oscillation with 72%, 52% and 65% respectively.

**Keywords:** Double pendulum crane; Linear matrix inequality (LMI); LMI region; State feedback control.

## 1. INTRODUCTION

An overhead double pendulum crane system is widely used in industries and construction sites [1]. This type of crane generates high undesirable sways at the hook and the payload thereby causing payload bouncing, twisting and swinging [2]. In addition, the crane acceleration and deceleration generate payload sways during starting and stopping operations. The force of inertia that acts on the payload when a command signal is injected to the crane also causes a high amplitude payload sways [3-4]. The aim to transport the payload from one point to another without or less payload sway to achieve precise positioning of the payload. Therefore, an effective control is required to achieve safety and effectiveness of crane operations [5]. Researchers proposed and presented different control approaches to solve the mentioned problems. Control strategies for the control of hook, payload and trolley position of the double pendulum crane have been presented in [6-17]. However, most of these references have presented controls that involved combination of two or three controllers for the control of trolley position, hook and payload.

Similarly, some control approaches were presented for a simple pendulum-like crane includes cascade control of two controllers [18-22]. Several other works on an optimal control of crane systems have been presented in [23-29]. For industrial applications, the Linear Quadratic Gaussian (LQG) control and the Model Predictive Control (MPC) are popular as they can guarantee closed-loop stability [30-32]. Some works presented artificial intelligent control [33-36]. Several other related works were robust control for an overhead crane [37-46]. However, most of the above-mentioned control schemes are made up of combination of two or three controllers. Mostly, three controllers were used for the control of double pendulum crane for the control of trolley, hook and payload. A single controller could be used to achieve similar performance. This work aims at reducing the number of the controllers to reduce the cost, complexity and size of the controlled system.

This paper proposes a robust LMI based state feedback controller for control of the double pendulum crane. Using this single LMI-based positive state feedback controller the trolley position, hook sway and payload sway can be controlled to achieve desirable performance. LMI are powerful design tools that have been used in areas of control engineering. This is because the stabilisation conditions can be formulated in terms of LMIs which can be efficiently solved using convex optimisation techniques. The control is designed by the selection of an LMI region in the stability plane where the poles of the crane are placed at a location within the region to achieve precise trolley positioning and low sways for both hook and payload.

## 2. DOUBLE PENDULUM CRANE MODEL DESCRIPTION

Figure 1 shows the schematic diagram of the double pendulum crane [47]. The parameters and variables  $m$ ,  $m_1$ ,  $m_2$ ,  $l_1$ ,  $l_2$ ,  $x$ ,  $\theta_1$ ,  $\theta_2$  and  $F$  represents the trolley mass, the hook mass, the payload mass, the cable length between the trolley and the hook, the cable length between the hook and the payload, the trolley position, the hook angle, the payload angle and the driving force respectively.

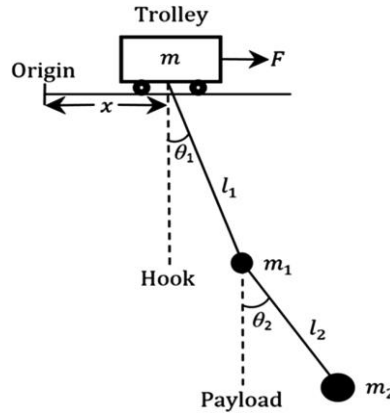


Figure 1. Schematic diagram of double pendulum crane

### 2.1 Dynamic Model

The dynamics of the double pendulum crane can be described by the following nonlinear equations [47]:

$$\ddot{x} = -\frac{l_1(m_1+m_2)(\dot{\theta}_1 \cos \theta_1 - \dot{\theta}_1^2 \sin \theta_1)}{(m+m_1+m_2)} - \frac{m_2 l_2 (\dot{\theta}_2 \cos \theta_2 - \dot{\theta}_2^2 \sin \theta_2)}{(m+m_1+m_2)} + \frac{F}{(m+m_1+m_2)} \quad (1)$$

$$\ddot{\theta}_1 = -\frac{\ddot{x} \cos \theta_1}{l_1} - \frac{m_2 l_2 (\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2))}{l_1(m_1+m_2)} - \frac{g \sin \theta_1}{l_1} \quad (2)$$

$$\ddot{\theta}_2 = -\frac{\ddot{x} \cos \theta_2}{l_2} - \frac{l_1 (\ddot{\theta}_1 \cos(\theta_1 - \theta_2) + \dot{\theta}_1^2 \sin(\theta_1 - \theta_2))}{l_2} - \frac{g \sin \theta_2}{l_2} \quad (3)$$

The parameters and variables of the system are shown in Table 1 [47].

Table 1. Parameters and the variables the system [47]

Parameter	Value /Unit
Trolley mass ( $m$ )	6.5 kg
Hook mass ( $m_1$ )	2 kg
Payload mass ( $m_2$ )	0.6 kg
Hook cable length ( $l_1$ )	0.53 m
Payload cable length ( $l_2$ )	0.4 m
Gravitational acceleration ( $g$ )	9.81 m/s <sup>2</sup>

### 2.2 Linear Model

For the purpose of designing the controller, the nonlinear model is linearised. The linearisation is done based on the assumption that  $\theta_1$  and  $\theta_2$  are very small. Therefore, the linearized equations can be obtained as:

$$\ddot{x} = -\frac{l_1(m_1+m_2)}{(m+m_1+m_2)} \ddot{\theta}_1 - \frac{m_2 l_2}{(m+m_1+m_2)} \ddot{\theta}_2 + \frac{F}{(m+m_1+m_2)} \quad (4)$$

$$\ddot{\theta}_1 = -\frac{1}{l_1} \ddot{x} - \frac{m_2 l_2}{l_1(m_1+m_2)} \ddot{\theta}_2 - \frac{g}{l_1} \theta_1 \quad (5)$$

$$\ddot{\theta}_2 = -\frac{1}{l_2} \ddot{x} - \frac{l_1}{l_2} \ddot{\theta}_1 - \frac{g}{l_2} \theta_2 \quad (6)$$

Using the parameters in Table 1, the state space description of the system can be obtained as:

$$\dot{x} = Ax + BU \quad (7)$$

$$y = Cx \quad (8)$$

where

$$x = [x \quad \dot{x} \quad \theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2]$$

$$U = F$$

$$y = [x \quad \theta_1 \quad \theta_2]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.9178 & 0 & 0.0013 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -31.4308 & 0 & 5.5461 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 31.8513 & 0 & -31.8513 & 0 \end{bmatrix}$$

$$B = [0 \quad 0.1539 \quad 0 \quad -0.2904 \quad 0 \quad 0]^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

### 3. LMI-BASED CONTROLLER

The proposed controller is a positive state feedback which is based on pole placement. The control of the crane is achieved by placing the closed loop poles of the system in a LMI region.

#### 3.1 Pole Placement in LMI Region

The stability and transient response of a linear system depends on the location of its poles in the complex plane. Consider a linear dynamic system:

$$\dot{X} = A_c X \quad (9)$$

The system in Equation (9) is said to be asymptotically stable if all its poles lie in the left-half plane. This is characterised in LMI terms by Lyapunov theorem, which can be stated as follows:

The system in Equation (9) is said to be asymptotically stable if there exist a real symmetric matrix  $P$  satisfying the following LMIs [2]:

$$A_c P + A_c^T P < 0, \quad P = P^T > 0 \quad (10)$$

The LMIs in Equation (10) gives the stability conditions for the system in Equation (9). An LMI region is a subset of the complex plane which is represented by a LMI [48]. Placing the systems poles in an LMI region may improve the transient performance of the system. LMI regions exist in different shape such as circular, sector, and horizontal strip among others. The LMI region considered in this work is shown in Figure 2. It is a combination of a circular and shifted left-half plane.

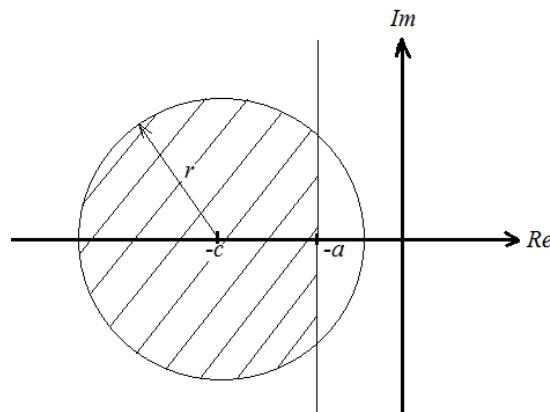


Figure 2: LMI region

The system in Equation (9) will have all its poles lying in the LMI region of Figure 2 if and only if there exists a symmetrical positive definite matrix  $P$  such that [48]:

$$A_c P + P A_c^T + 2aP < 0, \quad (11)$$

$$\begin{bmatrix} -rP & cP + P A_c^T \\ cP + A_c P & -rP \end{bmatrix} < 0, \quad P = P^T > 0 \quad (12)$$

The LMI in Equation (11) represents the shifted plane, while the LMI in Equation (12) represents the circle centered at  $c < 0$ , with radius  $r > 0$ .

### 3.2 Proposed Controller

Figure 3 shows the block diagram of the proposed control system, where  $X$  is the state vector,  $U$  is the control input,  $r$  is the reference input vector,  $K$  is the controller gain vector and  $N$  is the reference input scaling factor vector.

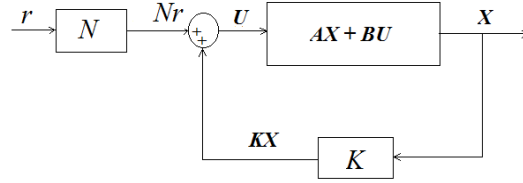


Figure 3. Proposed control system block diagram

From Figure 3, the control law can be written as:

$$U = KX + Nr \quad (13)$$

Substituting the proposed control law of Equation (14) in to the linear model of the crane system in Equation (9) gives:

$$\dot{X} = (A + BK)X + BNr \quad (14)$$

$$\dot{X} = A_c X + BNr \quad (15)$$

At steady state:

$$\dot{X} = 0 \quad (16)$$

$$X = X_{ss} \quad (17)$$

The control goal is to make the system states,  $X$  track the reference inputs,  $r$  at steady state. Thus;

$$r = X_{ss} \quad (18)$$

Substituting Equations (16)-(18) in Equation (14) gives:

$$0 = (A + BK)X_{ss} + BNX_{ss} \quad (19)$$

Hence,

$$BN = -(A + BK) \quad (20)$$

Pre-multiplying both sides of Equation (20) by  $B^T$  yields:

$$B^T BN = -B^T (A + BK) \quad (21)$$

Pre and post multiplying both sides of Equation (21) by  $(B^T B)^{-1}$  gives:

$$N = -(B^T B)^{-1} B^T (A + BK) \quad (22)$$

From Equations (13) and (14):

$$A_c = (A + BK) \quad (23)$$

Substituting Equation (23) in Equations (11) and (12) gives the following LMIs:

$$AP + PA^T + BM + M^T B^T + 2aP < 0, \quad (24)$$

$$\begin{bmatrix} -rP & cP + PA^T + M^T B^T \\ cP + AP + BM & -rP \end{bmatrix} < 0 \quad (25)$$

$$P = P^T > 0 \quad (26)$$

where

$$K = MP^{-1} \quad (27)$$

By solving the LMIs in Equations (24)-(26),  $P$  and  $M$  can be found. The controller gains  $K$  can then be obtained by substituting  $P$  and  $M$  in Equation (27). The reference input scaling factor  $N$  can be obtained using Equation (22). By solving the LMIs in Equations (24)-(26) with MATLAB toolbox using  $a = 1$ ,  $r = 5$  and  $c = 5$ , the controller parameter  $K$  and  $N$  were obtained as:

$$K = [-12.8625 \quad -20.0415 \quad 177.0767 \quad 44.0322 \quad -144.1631 \quad -9.8475]$$

$$N = [12.8625 \quad 20.0415 \quad -267.1590 \quad -44.0322 \quad 159.0717 \quad 9.8475]$$

#### 4. SIMULATION RESULTS

The performance of the proposed controller was investigated via simulations in Simulink. The simulation results are compared with those from the work in [47] which uses a combination of three Proportional Integral Derivative (PID) controllers. The parameters of the PID controllers as obtained in [47] are given in Table 2. The performance of the proposed controller was investigated in terms of position tracking as well as robustness due to payload variations.

Table 2. Parameters of PID control scheme [47]

PID controller	Parameter	Value
PID1 (for trolley position)	$K_p$	19.7443
	$K_i$	0.0046
	$K_d$	15.9720
PID2 (for hook oscillation)	$K_p$	0.9709
	$K_i$	29.5439
	$K_d$	7.2471
PID3 (for payload oscillation)	$K_p$	0.6627
	$K_i$	1.5400
	$K_d$	0.1484

##### 4.1 Tracking Performance

The simulation was carried out with the desired trolley position of 0.6 m and the desired swing angle of  $0^\circ$  for both hook and payload. Figures 4-7 show the simulation results. Both controllers tracked the trolley position as well as the swing angles as shown in Figures 4, 5 and 6. Table 3 summarizes the simulation results.

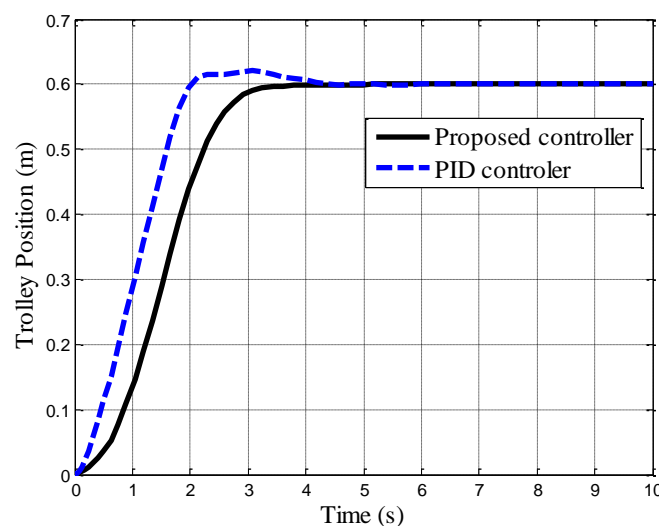


Figure 4: Trolley Position

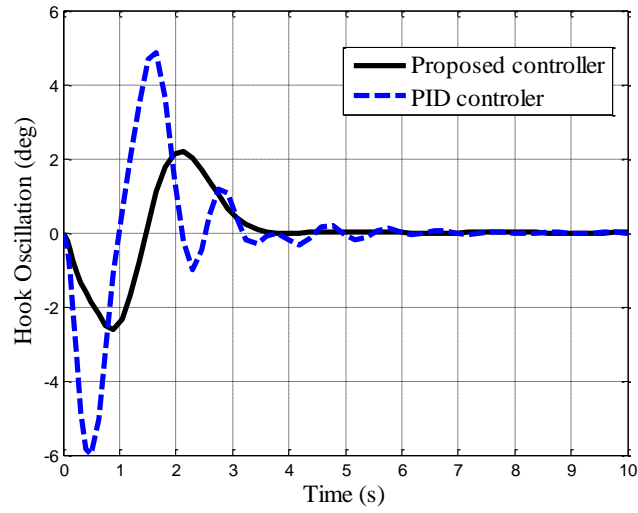


Figure 5: Hook swing angle

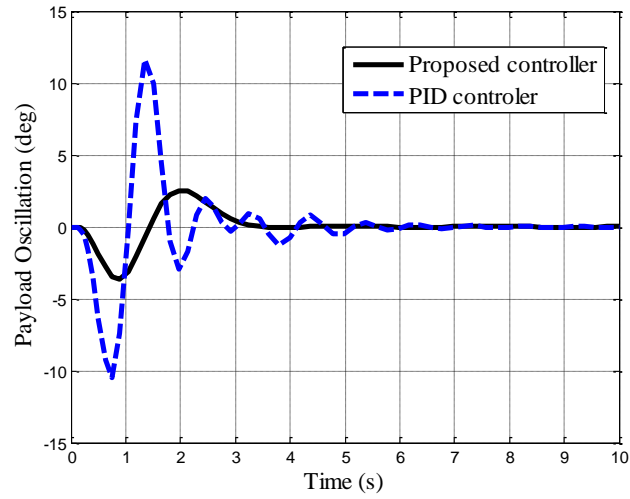


Figure 6: Payload swing angle

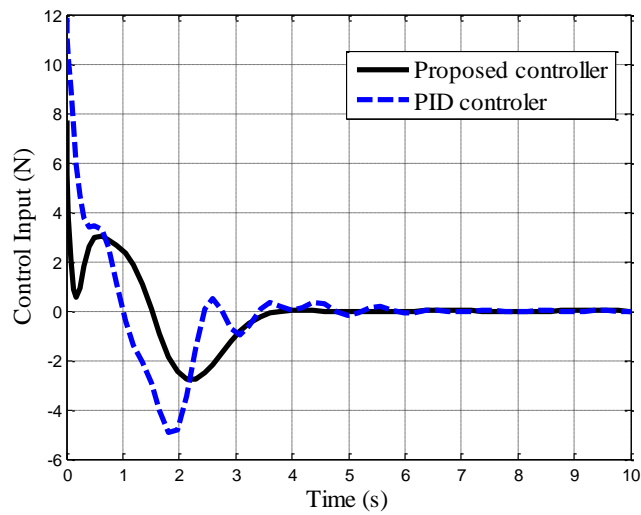


Figure 7: Control input

Table 3. Comparison of results

Controller	Trolley Position		Hook Oscillation		Payload Oscillation	
	$ST$ (s)	$OS$ (%)	$ST$ (s)	$\theta_{1max}$ (deg)	$ST$ (s)	$\theta_{2max}$ (deg)
Proposed	3.00	0.00	3.45	2.60	3.25	3.60
PID	3.53	3.50	5.37	5.99	6.57	11.61

$ST$ : Settling Time;  $OS$ : Overshoot

The proposed controller settled the trolley position in 3 seconds without an overshoot whereas the PID controller settled the tilt angle in 3.53 seconds with an overshoot of 3.5% as shown in Figure 4 and Table 3. The proposed controller damped out the hook oscillation in 3.45 seconds with a peak swing angle of  $2.6^{\circ}$  whereas the PID controller damped out the hook oscillation in 5.37 seconds with a peak swing angle of  $5.99^{\circ}$  as shown in Figure 5 and Table 3. The proposed controller damped out the payload oscillation in 3.25 seconds with a maximum swing angle of  $3.6^{\circ}$  whereas the PID controller damped out the payload oscillation in 6.57 seconds with a maximum swing angle of  $11.61^{\circ}$  as shown in Figure 6 and Table 3. From Figure 7, it can be seen that the initial control input required by the PID control scheme is high when compared to that required by the proposed controller.

#### 4.2 Effect of Payload Variation

The robustness of the proposed controller is investigated due to payload variation and the result is compared with that from the PID control scheme. The simulation was carried out with the payload mass doubled. Figures 8-11 show the simulation results and Table 4 summarizes the simulation results under this condition.

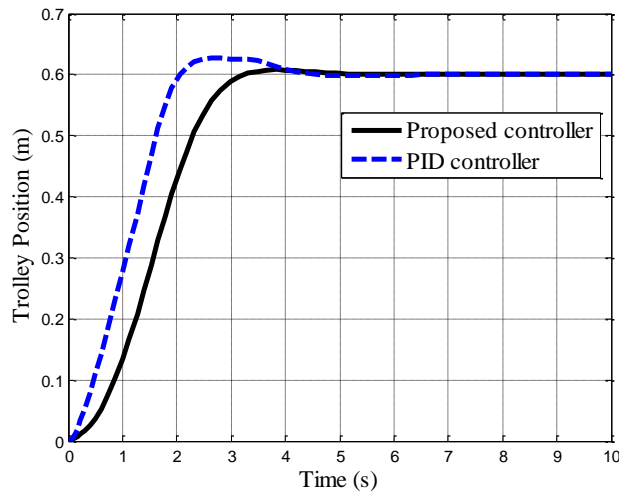


Figure 8. Trolley position with payload doubled

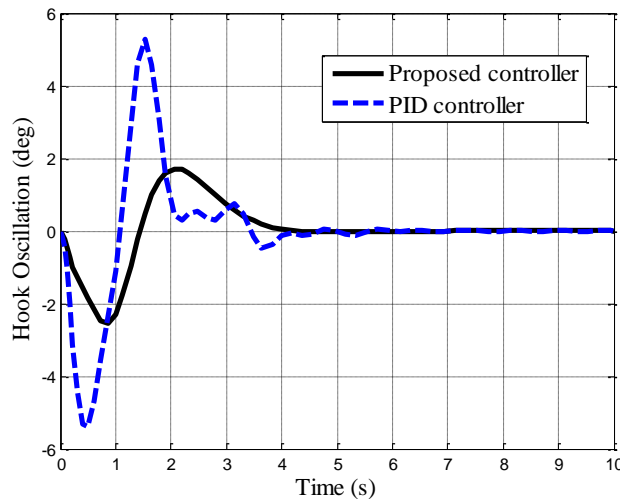


Figure 9. Hook swing angle with payload doubled

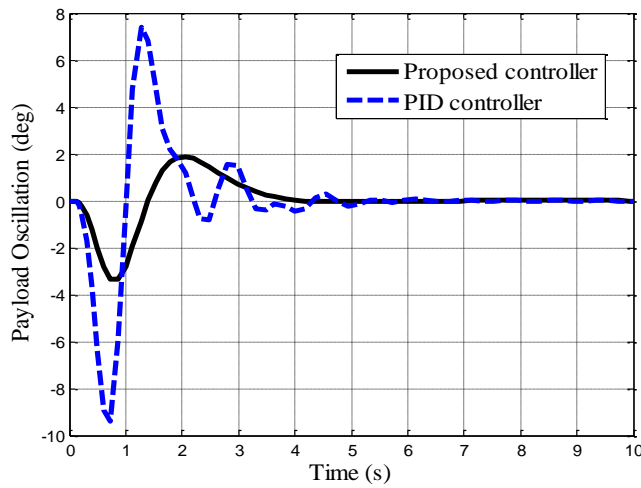


Figure 10. Payload swing angle with payload doubled

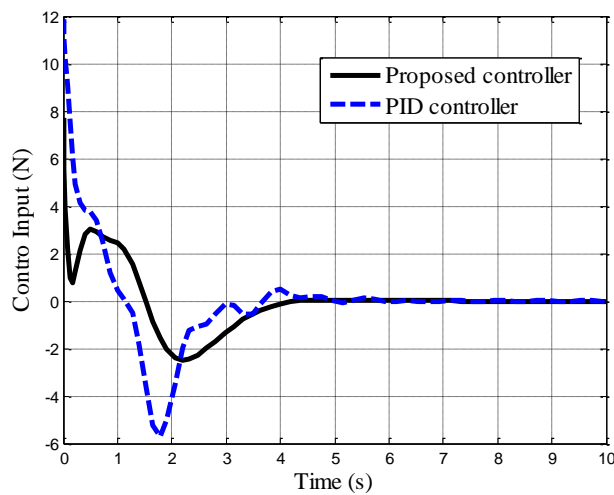


Figure 11. Control input with payload doubled

Table 4. Comparison of results with payload doubled

Controller	Trolley Position		Hook Oscillation		Payload Oscillation	
	<i>ST</i> (s)	<i>OS</i> (%)	<i>ST</i> (s)	$\theta_{1max}$ (deg)	<i>ST</i> (s)	$\theta_{2max}$ (deg)
Proposed	2.98	1.33	3.84	2.56	3.84	3.33
PID	3.53	4.68	5.33	5.37	5.14	9.39

*ST*: Settling Time; *OS*: Overshoot

With the proposed controller, the trolley position settling time reduces from 3 seconds to 2.98 seconds and the overshoot increases from 0% to 1.33% whereas, using the PID controller the settling time remains as 3.53 seconds, but the overshoot increases from 3.5% to 4.68% as shown in Figure 8 and Table 4. With the proposed controller, the hook oscillation damping time increases from 3.45 seconds to 3.84 seconds and maximum swing angle reduces from  $2.6^0$  to  $2.56^0$  whereas with the PID controller oscillation damping time reduces from 5.37 seconds to 5.33 seconds and the peak swing angle reduces from  $5.990^0$  to  $5.37^0$  as shown in Figure 9 and Table 4. With the proposed controller, the payload oscillation damping time increases from 3.25 seconds to 3.84 seconds and the maximum swing angle reduces from  $3.6^0$  to  $3.33^0$  whereas with the PID controller the payload oscillation reduces from 6.57 seconds to 5.14 seconds and the maximum swing angle reduces from  $11.61^0$  to  $9.39^0$  as shown in Figure 10 and Table 4. Figure 11 shows the control input under this condition. It can be seen that the initial control input required by the PID control scheme is high when compared to that required by the proposed controller.

## 5. CONCLUSION

An LMI based state feedback controller was proposed for the control of a double pendulum crane. The performance of the proposed controller was compared with that of a PID control scheme in terms of position tracking and robustness due to payload variation via simulations. The simulations results showed that both controllers tracked the trolley position and swing angles. However, the proposed controller performed better than the PID control scheme in terms of reducing the maximum swing angles, fast hook and payload oscillations damping and reducing the required initial control input.

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