

# Mathematical Model for Special Capacitated Vehicle Routing Problem Considering Environmental Factors

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**Abstract:** This research proposes a mathematical model for the capacitated vehicle routing problem with special operational constraints, incorporating environmental considerations in a real-world beverage distribution case study. The model determines the optimal selection of vehicles, routes, truck types, and loading configurations to minimize total carbon dioxide (CO<sub>2</sub>) emissions while satisfying practical capacity and operational requirements. The proposed model is formulated as a mixed integer linear programming problem and solved using the GUROBI Solver. Computational results indicate that the optimized routing plans achieve a minimum total emission of 145.70 kgCO<sub>2</sub>eq/L, outperforming the company's existing delivery strategy. Furthermore, experiments using daily demand data over a 10-day period demonstrate an average CO<sub>2</sub> emission reduction of 7.96%, implying improved fuel efficiency and more sustainable transportation operations. These findings highlight the practical relevance of applying exact optimization models to address current challenges in emission-aware logistics planning. Future research may extend the proposed framework to larger-scale networks, multi-objective formulations, and additional environmental indicators to further enhance decision-making in sustainable transportation systems.

**Keywords:** Carbon dioxide emissions; Green logistics; Mixed-integer linear programming; Sustainable transportation; Vehicle routing problem.

## 1. INTRODUCTION

The transportation industry is one of the most important sectors of the Thai economy, as Thailand serves as a transportation hub in the ASEAN (Association of Southeast Asian Nations) region. Operations in this industry must consider various cost factors including fuel, distance, labor, time, and fleet size to minimize expenses without compromising operational efficiency. Vehicle routing plays a crucial role in both efficiency and transportation costs, particularly for businesses distributing goods to retail stores, where fast and accurate deliveries are required. Currently, many companies emphasize environmentally friendly transportation to align with the Sustainable Development Goals (SDGs) and reduce environmental impacts such as greenhouse gas emissions, energy consumption, and pollution. In Thailand, several companies have adopted technologies such as GPS (Global Positioning System) systems and electric vehicles to plan routes that reduce costs and promote sustainability. However, vehicle routing in real-world operations still faces numerous challenges, such as uncertain customer demand, limited availability of vehicles and personnel, and insufficient vehicle capacity. In many distribution businesses, including the case study company Suprasert Karnka Co., Ltd., which delivers beverages to retailers in Chiang Mai, route planning is still performed manually and relies heavily on staff experience. Daily and unpredictable customer orders often require same-day deliveries, resulting in high variability in routes and imbalanced workloads. Combined with vehicle and manpower limitations, these issues frequently lead to inefficient routing, long travel distances, and human errors such as delivery delays or incorrect shipments. These inefficiencies not only reduce customer satisfaction but also increase transportation costs and impact profitability, highlighting the need for systematic route optimization.

To address these challenges, researchers have applied the Vehicle Routing Problem (VRP) framework, particularly the Capacitated VRP (CVRP), which incorporates vehicle capacity, and the Green VRP (GVRP), which emphasizes environmental impact reduction. The development of mathematical models and the use of optimization software such as the GUROBI Solver can help identify optimal routes, reduce costs, and enhance sustainability. The VRP has attracted significant research attention for several decades, particularly in the CVRP variant, which focuses on route planning under vehicle capacity constraints. Several studies have proposed diverse algorithms and models, including hybrid metaheuristics to reduce costs and environmental impacts [1], the integration of Fuzzy C-Means Clustering with 2E-CVRP-E (Two-Echelon Capacitated Vehicle Routing with Environmental consideration) to improve total cost and carbon dioxide (CO<sub>2</sub>) emissions [2], and Iterated Local

Search for large-scale F-CVRP (Family Capacitated Vehicle Routing Problem) [3]. Additionally, the concepts of multi-trip routing and sequence-based pallet loading have been applied to CVRP to account for vehicle loading efficiency and delivery schedules [4]. The Branch-Price-and-Cut Algorithm has been used to improve urban waste collection efficiency [5], while integrating CLP (Container Loading Problem) with CVRP through the 3L-PCVRP (The pickup capacitated vehicle routing problem with three-dimensional loading constraints) has helped reduce transportation costs using an improved Branch-and-Price Algorithm [6]. These studies demonstrate that Branch-and-Bound, Branch-and-Price, and metaheuristic techniques are effective tools for solving CVRP for both small- and large-scale problems.

For GVRP, research has focused on reducing environmental impacts. Hybrid ACO-VNS (Ant Colony Optimization-Variable Neighborhood Search) methods have been employed to reduce both economic costs and pollution [7], while exact algorithms have been used to reduce emissions and noise [8]. CNVRP (Carbon-Neutral Vehicle Routing Problem) and MSCNVRP (Multi-Stage Carbon-Neutral Vehicle Routing Problem) studies have also applied exact algorithms to reduce fleet size, total cost, and CO<sub>2</sub> emissions [9]. Furthermore, several studies have proposed strategies to improve transportation efficiency, reduce emissions, and support sustainability goals [10]-[14].

In addition to these approaches, recent studies have further extended the CVRP framework to better reflect real-world operational conditions. The incorporation of scheduled arrivals, split deliveries within time windows, and emission considerations has been shown to enhance operational flexibility while reducing transportation costs and carbon emissions [15]. Routing problems defined on road networks rather than customer nodes have been addressed through advanced local search strategies, demonstrating improved solution quality and reduced computational time for large-scale instances [16]. Moreover, the integration of three-dimensional loading constraints into CVRP formulations has enabled scalable solution approaches that better capture vehicle loading feasibility and improve solution quality for large problem instances [17].

Although numerous studies have proposed algorithms and heuristic approaches to address VRP and its variants, several research gaps remain. These include the limited consideration of daily customer demand, uncertainties in delivery schedules, dynamic inventory management, and the simultaneous integration of multiple operational and environmental factors, such as vehicle capacity, distance, time, and CO<sub>2</sub> emissions, within a single framework. Despite the increasing popularity of heuristic and metaheuristic methods for large-scale problems, Exact Algorithms remain essential because they guarantee optimality and serve as reliable benchmarks for validating approximate methods. Their precision and mathematical rigor are particularly valuable for small- to medium-sized instances, where they can generate high-quality solutions and provide insights into problem structures. Thus, employing Exact Algorithms not only ensures solution accuracy but also contributes to the development of more robust and efficient heuristic models in future research.

This research aims to develop a mathematical model that integrates CVRP and GVRP to improve route efficiency in daily beverage distribution operations by considering vehicle capacity, travel distance, and emissions. The model uses the GUROBI Solver to obtain optimal solutions that reduce costs and enhance sustainability, while incorporating daily customer demand and warehouse and personnel constraints. Therefore, the model addresses practical logistics challenges more effectively than previous studies. The main contributions of this study are summarized as follows.

- (a) This study proposes a Mixed Integer Linear Programming (MILP) formulation that integrates the CVRP and the GVRP while explicitly incorporating realistic operational constraints derived from daily beverage distribution practices.
- (b) Special loading and routing constraints are introduced to eliminate infeasible delivery patterns, such as unbalanced pallet unloading and excessive customer visits per trip, ensuring that the optimized routes are directly implementable in real-world operations.
- (c) An exact optimization-based solution approach using the GUROBI Solver is employed to guarantee optimality for small- to medium-scale routing problems, providing a reliable benchmark for future heuristic or metaheuristic developments.
- (d) The proposed model is evaluated through comprehensive computational experiments, including sensitivity analysis and multi-day testing using real demand data, to demonstrate robustness and environmental benefits.
- (e) A user-friendly interface is developed to support practical adoption by non-technical users, bridging the gap between theoretical optimization models and real-time operational decision-making.

In this study, the term simulation refers to a series of computational experiments conducted by repeatedly solving the proposed optimization model under different parameter settings, rather than to stochastic or discrete-event simulation. Specifically, the Capacitated Green Vehicle Routing Problem is formulated as a MILP model and solved using the GUROBI Solver for each experimental scenario. By systematically varying key parameters such as customer demand, fuel consumption rates, and travel distances, the model's robustness and performance are evaluated through multiple optimization runs while maintaining a consistent model structure.

In this study, the GUROBI Solver is employed as the core optimization engine due to its strong capability in solving MILP models with guaranteed optimality and high computational efficiency for small- to medium-scale problems. GUROBI is particularly suitable for this research because the proposed CVRP-GVRP formulation includes complex operational constraints and environmental considerations that require precise and reliable solutions. Rather than developing a fully stand-alone optimization program, GUROBI is used to complement the research framework by serving as a robust solver, while a separate user interface is developed to enhance accessibility and practical usability for non-technical users. This modular approach allows the optimization model, data inputs, and user interaction components to be flexibly managed and updated, supporting real-world decision-making without sacrificing solution quality. The remainder of this paper is organized as follows. Section 2 presents the research methodology, including the conceptual framework and mathematical model formulation. Section 3

reports the case study results and sensitivity analyses. Section 4 discusses the findings and implications of the proposed approach, and Section 5 concludes the paper with key insights and directions for future research.

## 2. RESEARCH METHODS

This research proposes a mathematical model to address the CVRP with environmental considerations, also known as the GVRP. The overall research process is structured into seven main stages, as illustrated in Figure 1. It begins with the conceptual model design and mathematical model formulation, followed by data collection from the case study. The formulated model is then solved using the GUROBI optimizer, and user tools are developed to support practical implementation. Subsequently, results and sensitivity analyses are conducted to evaluate the model's performance. Finally, the study concludes with key findings and implications derived from the research outcomes.

### 2.1 Proposed Mathematical Model

#### 2.1.1 Conceptual Model

After defining the research objectives, a comprehensive literature review on VRP, CVRP, and GVRP was conducted to identify appropriate solution techniques. Based on the review findings, this study adopts a combined modeling approach that integrates the CVRP which addresses vehicle capacity constraints with the GVRP, which incorporates environmental considerations through carbon emission calculations. The model is solved using an Exact Algorithm via the GUROBI Solver, as the problem size is relatively small and can be solved to optimality. The following assumptions are applied in this study:

- Vehicle capacities are fixed and known in advance.
- All customer demands are deterministic and must be fully satisfied within a single delivery.
- Each vehicle departs from and returns to the warehouse after completing its assigned route.
- Travel distances and fuel consumption rates are constant and based on real-world company data.
- Carbon emissions are directly proportional to fuel consumption.

These assumptions help simplify the model while maintaining its practical relevance to real-world operations.

#### 2.1.2 Mathematical Model

The proposed mathematical model represents the truck transportation system of the case study company, where each vehicle departs from a single distribution center, serves a set of retail stores, and returns to the same distribution center within a planning period. The objective is to determine the optimal selection of truck types and delivery routes that minimize environmental impact while satisfying operational and capacity constraints. The model is formulated as a MILP problem.

Let  $i$  and  $j$  denote origin and destination nodes, respectively ( $i, j = 1, 2, \dots, N$ ), where node 1 represents the distribution center and the remaining nodes represent retail customers. Let  $k = 1, 2, \dots, R$ , denote the trip index and  $m = 1, 2, \dots, T$  denote the truck type. The binary decision variable  $X_{ijkm}$  equals 1 if a vehicle of type  $m$  travels from node  $i$  to node  $j$  on trip  $k$ , and 0 otherwise. The continuous variable  $W_{ijkm}$  represents the load (in pallets) transported along arc  $(i, j)$  on trip  $k$  by truck type  $m$ . Model parameters include  $CAPmax_m$ , the maximum loading capacity of truck type  $m$ ;  $Demand_j$ , the demand of customer  $j$ ;  $DISTANCE_{ij}$ , the travel distance between nodes  $i$  and  $j$ ;  $RFC_m$ , the fuel consumption rate of truck type  $m$ ; and  $GHG$ , the greenhouse gas emission factor of diesel fuel.

The objective function is defined in Equation (1), which minimizes the total carbon dioxide emissions generated from transportation activities, calculated as the product of travel distance, vehicle fuel consumption rate, and the emission factor.

$$Max Z = \sum_i \sum_j \sum_k \sum_m X_{ijkm} \times DISTANCE_{ij} \times RFC_m \times GHG \quad (1)$$

The constraints of the model are grouped according to their functional roles to improve clarity and interpretation. Constraints (2) and (3) ensure route completeness by requiring that each vehicle departs from and returns to the distribution center exactly once for each trip.

$$\sum_{j>1} \sum_m X_{1jkm} = 1 \quad \forall k \quad (2)$$

$$\sum_{i>1} \sum_m X_{i1km} = 1 \quad \forall k \quad (3)$$

Constraints (4) and (5) guarantee that each customer node is visited exactly once during the planning horizon.

$$\sum_k \sum_{j \neq i} \sum_m X_{ijkm} = 1 \quad \forall i \neq 1 \quad (4)$$

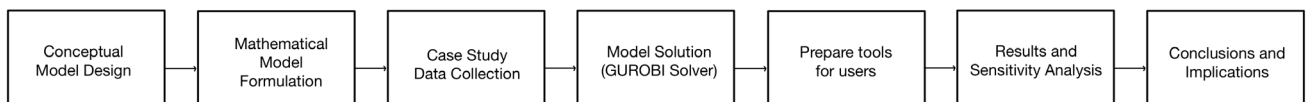


Figure 1. The overall framework of the research methodology.

$$\sum_k \sum_{i \neq j}^I \sum_m X_{ijkm} = 1 \quad \forall j \neq 1 \quad (5)$$

Constraints (6)-(9) represent flow conservation, demand satisfaction, and vehicle capacity restrictions. These constraints ensure that the quantity of goods delivered to each customer meets its demand without exceeding the vehicle capacity.

$$\sum_{i \neq j}^I W_{ijkm} - \sum_{i \neq j}^I W_{jikm} = DEMAND_j \times \sum_{i \neq j}^I X_{ijkm} \quad \forall j \neq 1, \forall k, m \quad (6)$$

$$W_{ijkm} \leq (CAPmax_m - DEMAND_i) \times X_{ijkm} \quad \forall i, j \text{ and } i \neq j, \forall k, m \quad (7)$$

$$W_{ijkm} \leq CAPmax_m \quad \forall i, j \text{ and } i \neq j, \forall k, m \quad (8)$$

$$W_{ijkm} \geq DEMAND_j \times X_{ijkm} \quad \forall i, j \text{ and } i \neq j, \forall k, m \quad (9)$$

Constraint (10) is a subtour elimination constraint that preserves route continuity and prevents disconnected cycles.

$$u_i - u_j + (N \times \sum_k X_{ijkm}) \leq N - 1 \quad \forall i \neq j; i = 2, 3, \dots, N; j = 2, 3, \dots, N, \forall m \quad (10)$$

Constraints (11) and (13)-(15) impose special operational restrictions for medium four-wheel trucks used in the case study. Constraint (11) limits the load transported between any two customer nodes to no more than two pallets, preventing unsafe or unbalanced unloading configurations.

$$W_{ijkm} \leq 2 \quad \forall i \neq 1, \forall j \text{ and } j \neq i, \forall k, \forall m = 1, \forall m = 2 \quad (11)$$

Constraint (12) enforces flow balance consistency across all routes.

$$\sum_{i \neq j}^I X_{ijkm} - \sum_{i \neq j}^I X_{jikm} = 0 \quad \forall j, k, m \quad (12)$$

Constraint (13) limits the number of customers visited per trip to at most three.

$$\sum_{j \neq i}^J \sum_i \sum_m X_{ijkm} \leq 3 \quad \forall k \quad (13)$$

Constraints (14) and (15) ensure that when a customer requires three or more pallets, the vehicle returns directly to the distribution center without serving additional customers.

$$\sum_{i \neq j}^I X_{ijkm} = X_{ijkm} \quad \forall k, m, \forall j \neq 1 \in \{DEMAND \geq 3\} \quad (14)$$

$$W_{j1km} = 0 \quad \forall k, m, \forall j \neq 1 \in \{DEMAND \geq 3\} \quad (15)$$

Overall, the constraints in Equations (11)-(15) ensure realistic and implementable routing behavior for medium four-wheel trucks by restricting full truckload deliveries to two feasible configurations: serving a single customer with four pallets or serving two customers with two pallets each. These constraints eliminate infeasible delivery patterns such as (3+1), (1+3), or (1+1+1+1), aligning the optimized solutions with actual operational practices observed in the case study company.

### 2.1.3 Solving the Model

After constructing the mathematical model, the next step is to obtain the optimal solution by implementing the model using a computer-based optimization tool. In this study, the model was developed and solved using the Python programming language in combination with the GUROBI Solver. The process begins with importing the required Python libraries, including those for mathematical modeling, data processing, and optimization. Subsequently, the problem data, decision variables, objective function, and all model constraints are defined within the Python environment. Once the model formulation is complete, the GUROBI Solver is executed to determine the optimal solution that minimizes total carbon dioxide emissions while satisfying all operational and capacity constraints. Finally, the results are extracted and presented to analyze the optimal transportation routes, vehicle assignments, and overall system performance obtained from the solution process.

To enhance transparency and facilitate understanding of the model's computational mechanism, Algorithm 1 presents a pseudocode description of the main procedures used in the Python-GUROBI implementation, including data input, model initialization, constraint formulation, optimization execution, and result extraction. This pseudocode provides a high-level overview of the solution workflow without excessive programming detail. In addition, Figure 2 illustrates a flowchart of the overall solution process, depicting the interaction between input data, MILP model formulation, the GUROBI optimization engine, and the output visualization through the User Interface. Together, Algorithm 1 and the flowchart clarify the logical structure of the solution process and support the reproducibility and practical implementation of the proposed model.

## Algorithm 1. Pseudocode for Solving the CVRP-GVRP Model

Input: Customer demand, distance matrix, truck capacities, fuel rates
Output: Optimal routes and truck assignments with minimum CO <sub>2</sub> emissions
Step 1: Initialize model using GUROBI
Step 2: Define decision variables ( $X_{ijkm}, W_{ijkm}$ )
Step 3: Define objective function (minimize CO <sub>2</sub> emissions)
Step 4: Add routing, capacity, flow conservation, and special constraints
Step 5: Execute GUROBI optimizer
Step 6: Extract optimal routes, loads, and emission values
Step 7: Visualize results via User Interface

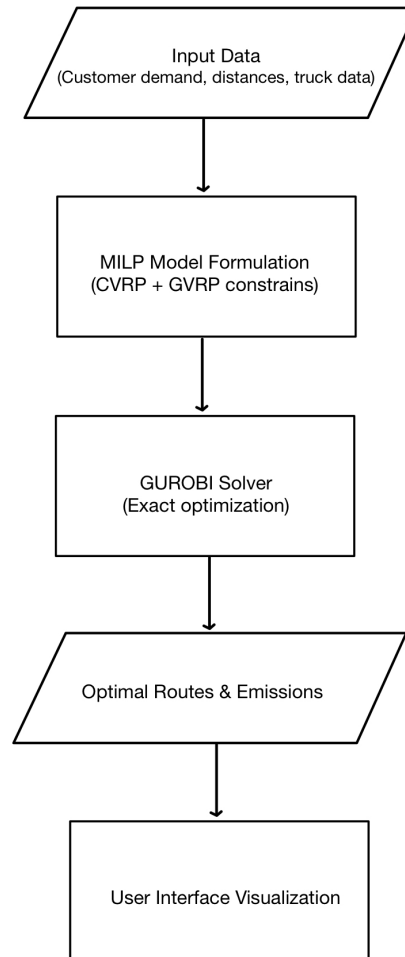


Figure 2. Flowchart of the proposed optimization framework.

### 3. RESULTS

#### 3.1 Case Study

The case study company, Suprasertkarnka Co., Ltd., is a beverage distribution business operating in Hang Dong District, Chiang Mai Province. The company distributes beverages, primarily supplying retail stores based on customer orders. Once orders are received, they are forwarded to the warehouse manager, who is responsible for planning delivery routes, preparing the goods, and coordinating transportation to each retail store as scheduled.

The company's operational data consist of one warehouse and twenty retail stores requiring deliveries. The delivery schedule includes sixteen trips per day, counted from the moment the delivery truck departs from the warehouse. The transportation network comprises four trucks of two different types: two small 4-wheel trucks, each with a maximum loading capacity of 2 pallets, and two large 4-wheel trucks, each capable of carrying up to 4 pallets. A critical operational constraint applies to the large 4-wheel trucks, which are open-sided vehicles. When fully loaded with 4 pallets, these trucks must maintain load balance; therefore, they cannot unload only 1 or 3 pallets at the first stop, as doing so would cause the vehicle to tilt, making further transportation unsafe.

The fuel consumption rates of the four trucks are 0.162, 0.158, 0.172, and 0.152 liters per kilometer, respectively, based on fuel usage records from 2024. The greenhouse gas emission factor for diesel fuel combustion in mobile sources is 2.7406 kgCO<sub>2</sub>eq/L, according to the Thailand Greenhouse Gas Management Organization (TGO). The daily demand for each retail store (node) varies depending on customer order requirements.

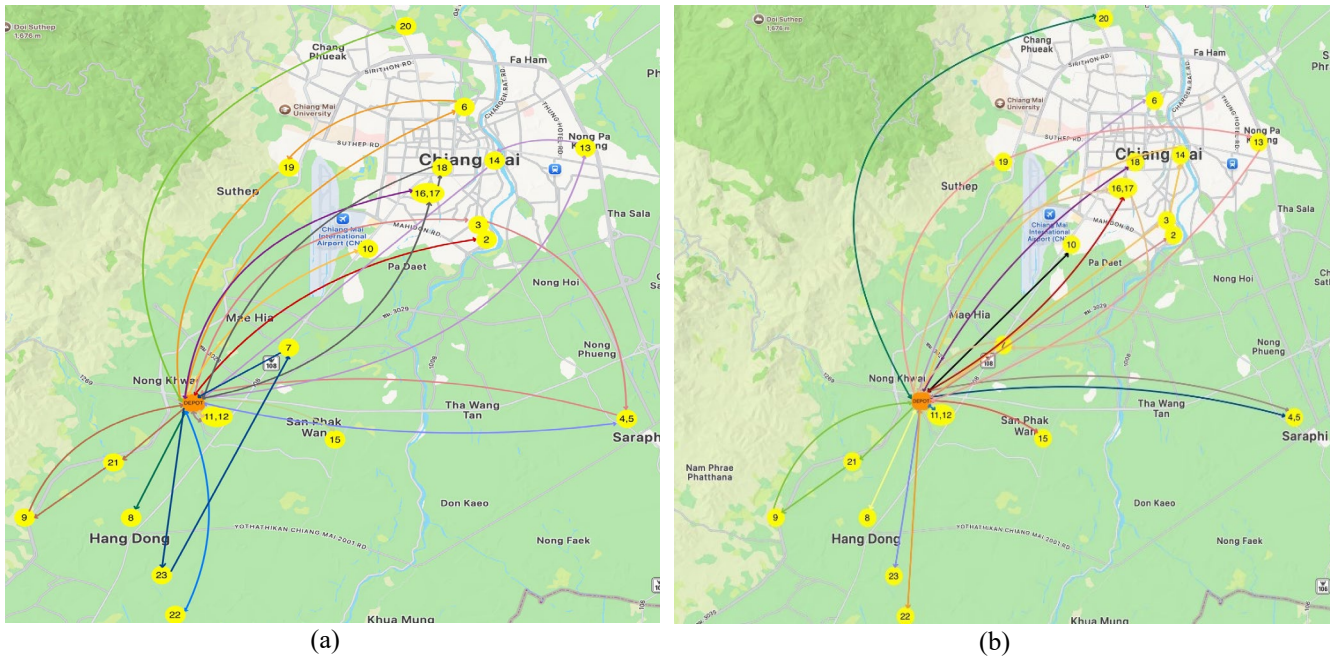


Figure 3. Comparison of vehicle routing plans: (a) optimized routes obtained from the computational model using the GUROBI Solver, and (b) the current routing plan used by the case study company.

Based on these data, the total carbon dioxide emissions generated by transportation activities are calculated from fuel consumption during each delivery trip. Specifically, the  $\text{CO}_2$  emissions for a given route are obtained by multiplying the travel distance (km) by the corresponding vehicle fuel consumption rate (L/km) and the diesel emission factor ( $\text{kgCO}_2\text{eq/L}$ ). The total emissions are then calculated by summing the emissions across all routes, vehicle trips, and truck types within the planning horizon. This calculation approach directly links routing decisions and vehicle selection to environmental impact and is consistent with standard practices in green vehicle routing and sustainable transportation studies.

### 3.2 Result from Mathematical Model

Based on the case study analysis using the GUROBI Solver, a computational experiment was conducted using one day of customer demand data. The results indicate that the minimum total  $\text{CO}_2$  emission achieved by the optimized routing model was  $145.70 \text{ kgCO}_2\text{eq/L}$ , with a total of 16 delivery trips required to satisfy all customer demands. The optimized routing plan is illustrated in Figure 3.

From Figure 3(a), it can be observed that the optimized model generates a total of 16 delivery trips, allowing all transportation tasks to be completed within a single day. In contrast, the company's original delivery plan required 18 trips over two days, as illustrated in Figure 3(b). The total carbon dioxide emissions from the existing routing plan were calculated at  $161.62 \text{ kgCO}_2\text{eq/L}$ , based on the objective function in which the total travel distance per trip is multiplied by the corresponding fuel consumption rate of each truck type and the emission factor of diesel fuel. In comparison, the proposed mathematical model effectively reduces the number of delivery trips by two and achieves a reduction in total carbon dioxide emissions of approximately  $15.92 \text{ kgCO}_2\text{eq/L}$  per day, equivalent to  $477.60 \text{ kgCO}_2\text{eq/L}$  per month and  $5,731.20 \text{ kgCO}_2\text{eq/L}$  per year.

### 3.3 Sensitivity Analysis

#### 3.3.1 Varying Parameter Conditions

To evaluate the stability and reliability of the proposed model, a sensitivity analysis was conducted by varying key parameters that influence  $\text{CO}_2$  emissions, namely the rate of fuel consumption (RFC), product demand, and transportation distance. The results, as shown in Table 1, indicate that when the RFC was reduced by 5%, total  $\text{CO}_2$  emissions decreased to  $138.52 \text{ kgCO}_2\text{eq/L}$ , demonstrating that the model is sensitive to fuel efficiency and that improving truck fuel performance can significantly mitigate environmental impacts. Conversely, increasing the RFC by 5% resulted in  $\text{CO}_2$  emissions rising to  $153.24 \text{ kgCO}_2\text{eq/L}$ , confirming that inefficient fuel usage amplifies environmental effects. Furthermore, when product demand decreased by one unit,  $\text{CO}_2$  emissions declined to  $125.35 \text{ kgCO}_2\text{eq/L}$ , whereas an increase of one unit in demand resulted in emissions of  $188.45 \text{ kgCO}_2\text{eq/L}$ . This reflects a positive correlation between product demand and emission levels, highlighting that fluctuations in demand substantially affect the sustainability of transportation operations. Additionally, a random increase in transportation distance elevated  $\text{CO}_2$  emissions to  $164.54 \text{ kgCO}_2\text{eq/L}$ , while a random decrease in distance reduced emissions to  $143.19 \text{ kgCO}_2\text{eq/L}$ , emphasizing the importance of efficient route planning in minimizing pollution. Overall, the analysis revealed that all modified parameters directly influence greenhouse gas emissions, with demand and distance being the most impactful factors. Therefore, effective demand management and optimized route planning are essential measures for reducing  $\text{CO}_2$  emissions and supporting sustainable transportation.

Table 1. Carbon dioxide emissions under varying parameter conditions.

Parameter	Change Condition	CO <sub>2</sub> Emission (kgCO <sub>2</sub> eq/L)	Impact Trend
RFC	Decreased by 5%	138.52	CO <sub>2</sub> decreases
	Increased by 5%	153.24	CO <sub>2</sub> increases
Demand	Decreased by 1 unit	125.35	CO <sub>2</sub> decreases
	Increased by 1 unit	188.45	CO <sub>2</sub> increases
Distance	Randomly increased	164.54	CO <sub>2</sub> increases
	Randomly decreased	143.19	CO <sub>2</sub> decreases

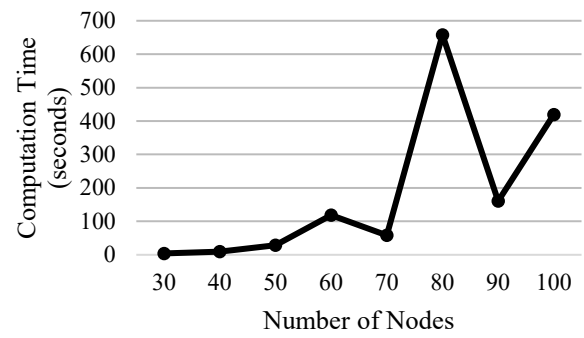


Figure 4. computation time with increasing numbers of customer nodes.

### 3.3.2 Time Computation Analysis

A sensitivity analysis was conducted to evaluate the performance and computational stability of the proposed model by increasing the number of vehicle trips, customer demand quantities, and customer nodes. The objective was to assess the impact of problem complexity on computation time as the problem size expanded. In the experimental setup, customer demand and inter-node distances were randomly generated to simulate realistic and uncertain operating conditions. The randomization followed these criteria: (1) customer demand values were randomly assigned within a range of 1-4 units (pallets) to represent varying customer requirements under vehicle capacity constraints, and (2) inter-node distances were randomly assigned within a range of 5-30 km to reflect the actual transportation radius from the distribution center in the case study. This randomized data generation approach allowed the model's performance to be evaluated independently of any specific dataset while effectively capturing increasing levels of problem complexity. The results, illustrated in Figure 4, show that computation time did not increase monotonically with the number of customer nodes. For example, when the number of nodes increased to 60, the solution time was 118.20 seconds; however, with 70 nodes, it decreased to 58.30 seconds. Similarly, when the number of nodes increased to 80, the computation time rose to 658 seconds but then dropped to 161.10 seconds at 90 nodes. These findings suggest that computation time is influenced not only by the problem size but also by the structural complexity and configuration of the constraints. In some cases, increasing the number of nodes may improve the model's decision efficiency, thereby reducing computation time.

### 3.3.3 Different Objective Analysis

The modification of the model's objective function in this section aims to conduct a sensitivity analysis examining how changes in the objective influence the model's outcomes in terms of efficiency and environmental impact. The comparison is made among three objective functions: (1) the objective proposed in this research, (2) the objective of minimizing total travel distance, and (3) the objective of minimizing total labor cost.

For the objective function of minimizing labor cost, the total cost is calculated by multiplying the decision variables by the labor cost associated with each vehicle type, as expressed in the following equation:

$$\sum_i \sum_j \sum_k \sum_m X_{ijkm} \times COST_m \quad (16)$$

where  $COST_m$  represents the fixed labor cost, derived from the incentive payment (bonus) per delivery trip for each vehicle type.

For the objective function of minimizing total travel distance, the model calculates the objective value using the following equation:

$$\sum_i \sum_j \sum_k \sum_m X_{ijkm} \times DISTANCE_{ij} \quad (17)$$

The researcher's proposed objective seeks to optimize vehicle utilization across all vehicle types by balancing shipment volumes and minimizing the use of vehicles with high energy consumption, thereby achieving the lowest possible carbon dioxide emissions. In contrast, the shortest-distance objective leads the model to prioritize the use of vehicles with the largest capacity (small 4-wheel trucks) to minimize the total distance traveled. Meanwhile, the minimum-cost objective results in the highest level of CO<sub>2</sub> emissions, as the model predominantly selects the lowest-cost vehicle type (medium 4-wheel trucks) for all deliveries. The comparative results of these objective functions are illustrated in Figure 5(a).

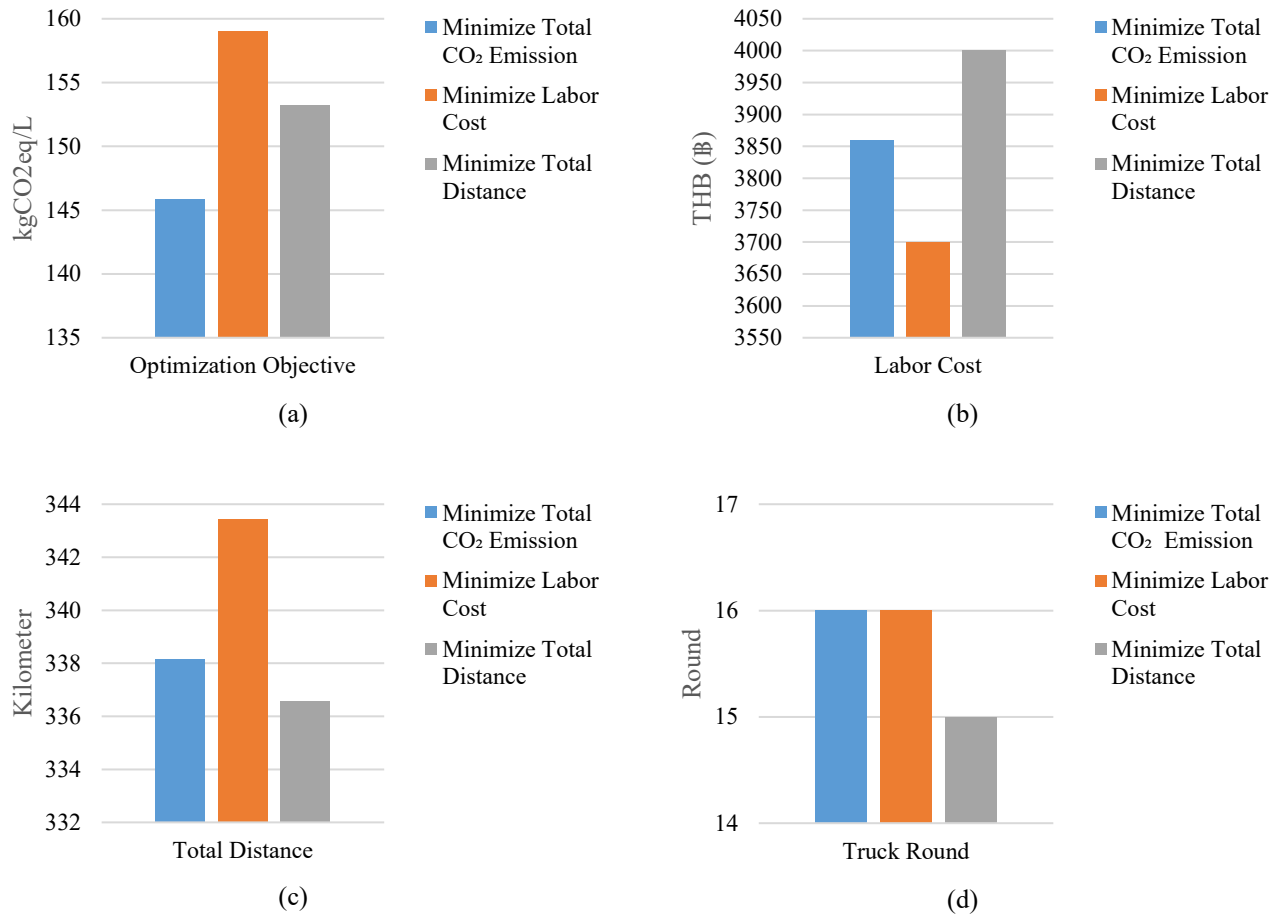


Figure 5. Sensitivity analysis results showing: (a) Total CO<sub>2</sub> emissions obtained under different optimization objectives; (b) Total labor cost for each objective function; (c) Total delivery distance corresponding to each objective; (d) Total number of vehicle trips under each optimization objective.

Specifically, the medium 4-wheel truck incurs a labor incentive of 100 THB per trip, while the small 4-wheel enclosed truck incurs 80 THB per trip. Since the medium 4-wheel truck requires more delivery staff compared to the other truck type, the model consequently prioritizes the use of small 4-wheel trucks to reduce total labor expenses. Although the total number of delivery trips remains unchanged, the routing pattern and vehicle selection differ to minimize labor costs. The comparative results of this objective are illustrated in Figure 5(b).

This objective seeks to determine the shortest possible routes, thereby reducing the total number of delivery trips and alleviating the overall transportation workload. However, minimizing distance alone does not necessarily guarantee a reduction in CO<sub>2</sub> emissions or labor costs. As illustrated in Figure 5(c), the model effectively identifies shorter routes, but trade-offs may occur in other performance aspects. Furthermore, as shown in Figure 5(d), a comparative analysis reveals that changing the model's objective function significantly influences the number of trips and the resulting objective values. Although each objective leads to distinct planning strategies, the practical outcomes—particularly in terms of total trips and delivery time—remain relatively comparable. Therefore, the choice of objective should align with organizational policies or managerial priorities, depending on which performance aspect is considered most critical.

### 3.3.4 Multi-Case Experiment

To evaluate the performance of the proposed model under real-world operating conditions, daily product demand data from the case study company over a 10-day period were used to compare the company's existing transportation routes (Current Route) with the optimized routes generated by the proposed model (Optimized Route). The primary performance indicator was the total CO<sub>2</sub> emissions (kgCO<sub>2</sub>eq/L), which corresponds to the model's objective function. The percentage change in CO<sub>2</sub> emissions between the two routing strategies was calculated using the following equation:

$$\% \Delta CO_2 = \frac{CO_{2Current} - CO_{2Model}}{CO_{2Current}} \times 100 \quad (18)$$

where  $CO_{2Current}$  denotes the total CO<sub>2</sub> emissions from the company's existing routing plan and  $CO_{2Model}$  represents the total CO<sub>2</sub> emissions obtained from the optimized model. The results of this comparison are summarized in Table 2.

Table 2. Comparison of daily delivery performance between the actual operation and the proposed optimization model based on real demand data from the case study company (10 days).

Day	No. of Customers	Actual Operation		Proposed Model		Computation Time (min)	% Reduction
		CO <sub>2</sub> (kgCO <sub>2</sub> eq/L)	No. of Trips	CO <sub>2</sub> (kgCO <sub>2</sub> eq/L)	No. of Trips		
1	14	116.43	12	105.66	10	0.01	9.25
2	15	122.67	13	111.63	11	0.01	9.00
3	10	70.82	8	65.52	7	0.01	7.48
4	13	111.67	9	102.56	10	0.01	8.16
5	16	125.28	13	114.62	11	0.01	8.51
6	8	54.16	7	51.59	6	0.01	4.75
7	18	146.88	16	132.51	14	2.10	9.78
8	12	108.78	11	99.49	9	0.01	8.54
9	17	125.72	14	116.68	12	0.30	7.19
10	13	77.26	9	71.88	8	0.01	6.96
<b>Average</b>	<b>13.60</b>	<b>105.97</b>	<b>11.20</b>	<b>97.21</b>	<b>9.80</b>	<b>0.248</b>	<b>7.96</b>

The experimental results over the 10-day period revealed that the proposed model successfully reduced carbon dioxide emissions on all observed days, with reductions ranging between 4% and 10%, and an average reduction of 7.96% compared to the company's existing delivery routes. A paired t-test was conducted to statistically validate the superiority of the proposed model. The results indicate a significant reduction in CO<sub>2</sub> emissions compared with the current routing strategy ( $t = 8.05$ ). Therefore, the improvement achieved by the proposed optimization model is statistically significant and unlikely to be due to random variation. This demonstrates the model's ability to adapt effectively to fluctuations in daily product demand while enhancing the overall sustainability of transportation operations.

### 3.4 User Interface

When implementing the developed model in practice, certain usability challenges may arise because some users are not familiar with the Python programming language. This limitation can make the operation process more complex and less efficient than intended. Therefore, it is necessary to improve the system's usability by developing a User Interface (UI) that simplifies user interaction and enhances accessibility. The proposed UI enables users to conveniently modify demand values and view the results without requiring any programming knowledge. By simply entering numerical values into the input fields, the system automatically computes and displays the optimized results in real time, thereby reducing complexity and improving operational efficiency, as illustrated in Figure 6.

The front-end interface is developed using Next.js, a framework built on React, which facilitates the creation of dynamic and responsive web applications. The user interface is designed with input fields and interactive buttons, implemented using HTML, to collect user-defined parameters. The input data are then managed as state variables using JavaScript, which subsequently communicates with the back-end API. This API functions as a bridge, transmitting the input data to the mathematical optimization model for computation. The resulting optimal solutions are processed and returned through JavaScript, which dynamically renders and visualizes the output within the user interface. The overall data flow and interaction between system components are illustrated in Figure 7.

## 4. DISCUSSION

Previous research on the CVRP and the GVRP has demonstrated that incorporating operational and environmental constraints can significantly improve transportation efficiency and sustainability [1], [3]. Capacity-aware routing and environmental considerations have been shown to lead to measurable reductions in cost and emissions. Metaheuristic and multi-objective approaches have been widely applied to large-scale GVRP instances [7], [12], while exact formulations have been shown to remain computationally feasible for well-structured and medium-sized problems [4], [5]. Collectively, these studies provide a strong methodological foundation for integrating environmental objectives into vehicle routing models.

Building on this foundation, the results obtained in this study demonstrate that the proposed MILP-based CVRP-GVRP model can effectively reduce total carbon dioxide emissions while maintaining operational feasibility under real-world constraints. The optimized routing plans generated by the model reduced unnecessary travel distance, improved vehicle utilization, and eliminated infeasible loading and unloading patterns that commonly arise in manual planning. Compared with the company's existing routing practice, the proposed approach achieved consistent reductions in CO<sub>2</sub> emissions across multiple demand scenarios, confirming its robustness and practical applicability for daily distribution operations.

A notable strength of the proposed model lies in its ability to explicitly capture realistic operational constraints, particularly those related to pallet loading stability in medium four-wheel trucks. By restricting infeasible delivery configurations such as uneven pallet unloading or excessive customer visits per trip, the model ensures that optimized routes are not only mathematically optimal but also practically implementable. This alignment between optimization outcomes and real operational behavior enhances the credibility of the model and supports its adoption in practice.

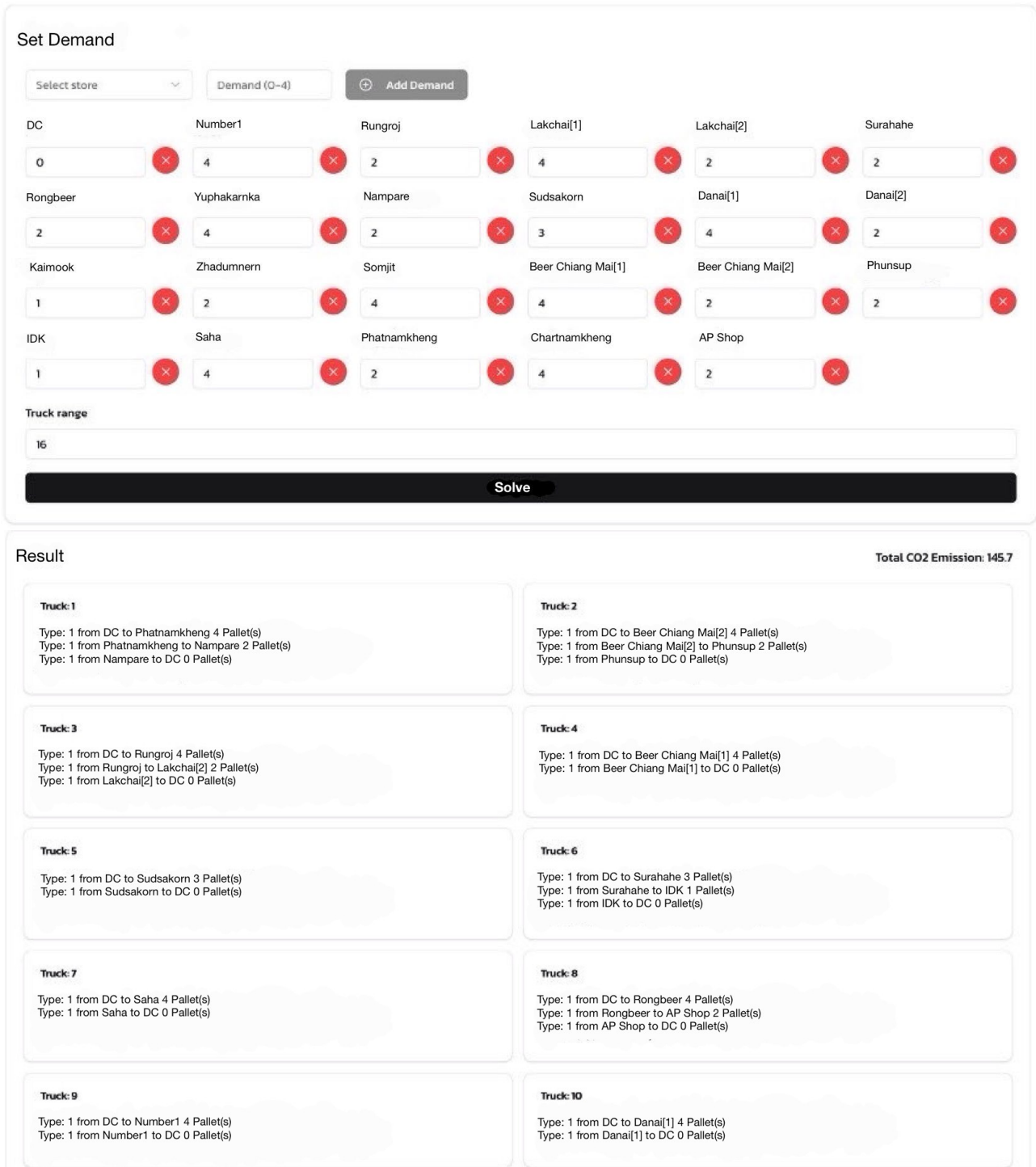


Figure 6. The User interface layout illustrates the configuration of demand and truck range parameters, along with the visualization of the optimization results.

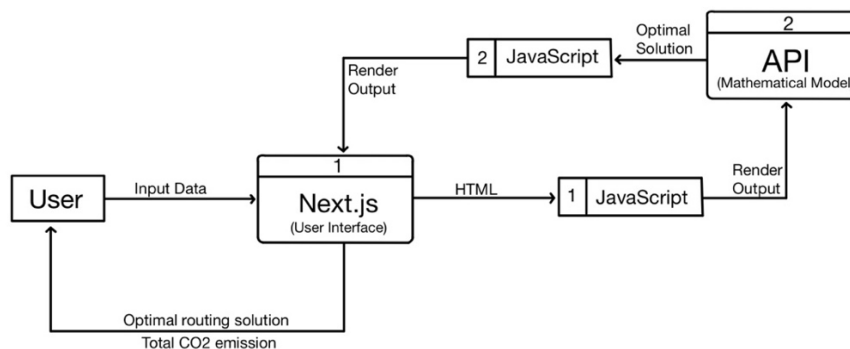


Figure 7. Data flow diagram of the UI system.

Another key contribution of this research is the development of a UI that enables practitioners to interact with the optimization model without requiring programming expertise. The UI allows users to modify demand levels and vehicle parameters and to generate optimized routing solutions in real time. This feature is particularly valuable in operational environments characterized by daily demand fluctuations and time-sensitive decision-making. By bridging the gap between theoretical optimization and practical implementation, the proposed framework enhances accessibility and supports data-driven decision-making in small- and medium-sized logistics operations.

Despite these promising results, several limitations of the study should be acknowledged. First, the environmental assessment focuses exclusively on CO<sub>2</sub> emissions and does not explicitly account for other pollutants such as nitrogen oxides (NO<sub>x</sub>), particulate matter (PM), or noise emissions. Second, while the use of an exact optimization approach ensures solution optimality, its computational scalability may be limited for large-scale routing problems involving a high number of customers or vehicles. Additionally, the model was validated using data from a single case study company, which may limit the generalizability of the findings.

## 5. CONCLUSION

This study developed a mathematical model to solve the CVRP and GVRP, taking into account vehicle capacity constraints and environmental considerations. The model aimed to minimize both transportation costs and carbon dioxide emissions by incorporating operational parameters such as distance, fuel consumption rate, and vehicle load capacity. Optimization was performed using the GUROBI Solver, which efficiently identified the optimal set of delivery routes under real-world constraints. The results from the case study indicate that the proposed model successfully reduced total transportation emissions to 145.70 kgCO<sub>2</sub>eq/L through 16 optimized trips, leading to improved route balance, reduced fuel consumption, and enhanced operational efficiency compared with traditional manual routing.

Furthermore, a UI was developed to enable users to dynamically modify demand, truck range, and other parameters in real time, facilitating practical use for logistics planners without requiring programming expertise. This interactive component makes the system adaptable to fluctuating operational conditions, thereby bridging the gap between theoretical modeling and practical implementation.

Although the results demonstrate the effectiveness of the proposed model, several directions for future research remain. Computational scalability could be enhanced by integrating hybrid or metaheuristic algorithms to address larger-scale problem instances [18], as suggested by recent studies on advanced local search and scalable routing frameworks [16]. Additional validation using datasets from multiple companies and industrial contexts would further strengthen the model's generalizability and robustness. Future extensions may also incorporate multiple environmental indicators, such as NO<sub>x</sub> and particulate matter (PM), and adopt multi-objective formulations that balance economic and environmental performance, in line with recent CVRP extensions that integrate emission considerations and operational flexibility [19]. Moreover, extending the model to include stochastic demand, time windows, and multi-echelon supply chain structures would improve realism and applicability. Finally, the integration of artificial intelligence or fuzzy-based decision-support techniques could enhance model adaptability and support decision-making under uncertainty [20]-[21].

In conclusion, the proposed optimization framework and user interface collectively contribute to a more sustainable, efficient, and intelligent logistics system. The model offers a practical decision-support tool that promotes environmental responsibility while maintaining operational feasibility, serving as a solid foundation for further research and industrial application.

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## DECLARATION OF CONFLICTING INTERESTS

The authors declare the following potential conflicts of interest: This research was conducted in collaboration with Suprasertkarnka Co., Ltd., which provided relevant operational data and logistical information used in the case study.

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