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Vibration Analysis of a Jib Crane using Frame Structures Approach

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Abstract: The vibration analysis of a jib crane was investigated using the method of vibration analysis of frame structures. This approach is theoretically based in order to derive the governing equations of vibration for the system. The jib crane was treated as a frame structure with the mast and jib taken as independent frame structures. The resulting frame structures were then treated as beams that conform to Euler-Bernoulli beams in order to analyse their vibration properties. Amplitude of vibration, the first three natural frequencies as well as the first three mode shapes were calculate using this approach as well as the response of the system. The theoretical approach was derived to accommodate all standard sets of natural frequencies and mode shapes of the jib crane from the established boundary conditions. This was developed by an assumed mode method and incorporated a single mode expansion for the developed system. The effect of the load carried by the jib crane was taking into consideration which constitutes a harmonic force on the jib with addition of moment of this force to the mast. The resulting theoretical general equations from the single mode approach were then used to evaluate the vibration parameters of the jib crane using numerical values.

Keywords: Frame structures; Jib; Jib crane; Mast; Vibration.

1. INTRODUCTION

Different approaches were adopted in the modelling and analysis of different type of cranes in order to investigate the vibration parameters of such systems. The method employed in analysing such systems is crucial to the outcome of the investigation with respect to each vibration parameter. A static, modal and harmonic analysis of a column mounted jib crane using ANASYS software was presented in [1-2]. The study initially modelled a column mounted jib crane using CATIA and then imported the model into ANASYS for the static analysis of the jib. Also, a nonlinear model of the jib crane considering the simultaneous rotational motions of the boom and the post was studied. The crane nonlinear model and the developed feedback control scheme were simulated using the data of an actual jib crane as presented in [3]. Hadžikadunić *et al.* [4] presented a comparative analysis of several concept solutions and selected solutions of jib cranes, respectively using the KOMIPS system and the static-dynamic analysis of complex configuration while Ramli *et al.* [5] gave a comprehensive review on the control strategy for crane system.

However, these researches were presented without analysing the background modelling and analytical approach of the systems. Models resulting from the assumption of two methods of modelling the dynamic systems of truck crane namely, discrete model built by applying MES, and a theoretically worked out discrete-continuous model were presented in [6]. In addition, a theoretical work of deriving mathematical equations of vibration under certain conditions was presented in [7]. The study analysed the natural frequency and the causes of resonance of the system. These analyses were verified experimentally in real-time using a portable crane.

Moreover, a comparative analysis of expressions for analytical calculation of deflection of the jib was carried out [8]. The study derived the theoretical equations then applied them to the deflection of the tip of the jib. A theoretical approach was used in driving the vibration of Euler-Bernoulli beams in [9]. A point-loaded cantilever beam with a damped dynamic vibration absorber attached to the beam at desired locations was modelled and derived. This is crucial for the development of analytic solution as a basis for comparison of such systems.

In this paper, we present an application of vibration analysis of frame structures for the analysis of jib crane vibration. As most works researched on the FEM in analysing the cranes, we will derive the theoretical background for the governing equations of motion of a jib crane vibration. This modelling approach as motivated by Lin and Ro [10] where the frames are taken be to continuous structures, will be used in driving the necessary vibration equations of the jib crane.

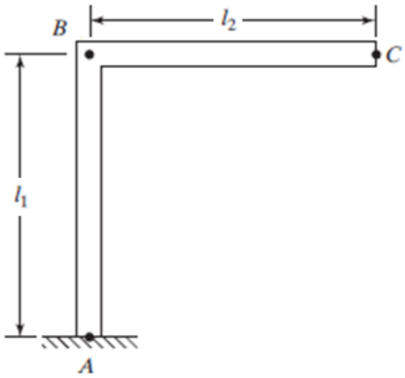


Figure 1. Schematic of the jib crane

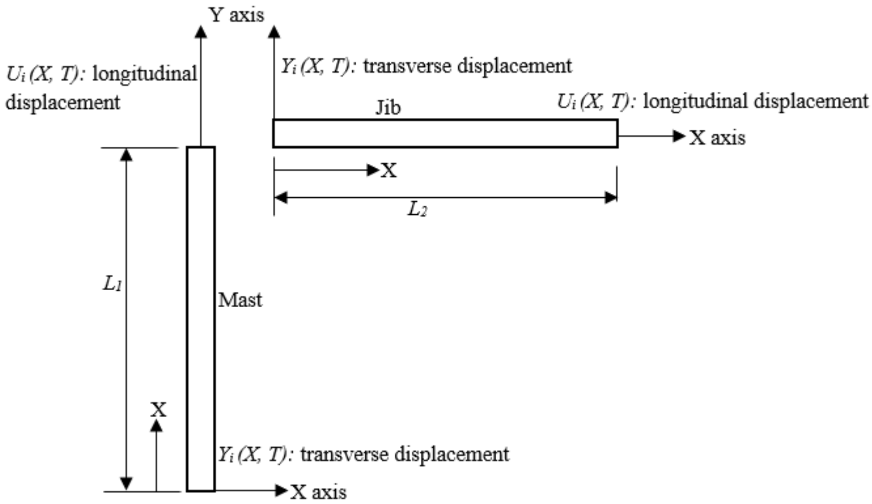


Figure 2. Transverse and longitudinal motion of the system

2. METHODOLOGY

2.1 Modelling a Jib Crane as a Frame Structure

The jig crane schematic is represented in Figure 1 with k frame and angle θ_1. The jib crane is sectioned into $k+1$ component at an angle position to enable it to be treated as a sub structural frame. The jib and mast with length l_1 and l_2, the position of the frame angle is located at *B*. Each of the jib and mast analysed by its transverse and longitudinal motion.

The jib crane is divided into two segments with the jib and mast representing independents segments that can be treated as beams. The *X* and *Y* axes represent the longitudinal and transverse displacements of the jib as shown in Figure 2, while conversely for the mast, the longitudinal and transverse displacements are represented by the *Y* and *X* axes respectively. Thus, vibration theories for Euler-Bernoulli beam and axial vibration of a rod are employed. The amplitude of vibration of the transverse and longitudinal displacements of the jib are denoted by $Y_i(X, T)$ and $U_i(X, T)$. The total length of the jib crane is $L = (L_1 + L_2)$. The equation of motion for each of the jib and mast with uniform cross-section, are:

Transverse motion:

$$EA\frac{\partial^4 Y_i(x,t)}{\partial x^4}+\rho A\frac{\partial^2 Y_i(x,t)}{\partial t^2}=0, X_{i-1}<X<X_i, i=1,2\ (k+1). \tag{1}$$

Longitudinal motion:

$$E\frac{\partial^2 U_i(x,t)}{\partial x^2}+\rho\frac{\partial^2 U_i(x,t)}{\partial t^2}=0, X_{i-1}<X<X_i, i=1,2\ (k+1). \tag{2}$$

The boundary conditions at point *A* is $W(0)=0$, $W'(0)=0$, $U(0)=0$ and $U'(0)$, At point *C* is $W''(l)=0$ and $W'''(l)=0$ and at point *B* is $Y_2(x) = U_1(x)$, $U_2(x) = -Y_1(x)$, $Y_2'(x) = Y_1'(x)$, $Y_2''(x) = Y_1'''(x)$, $Y_2'''(x) = -U_1'''(x)$, $U_2^p(x) = Y_1^p(x)$.

This compatibility conditions at point *B* are

$$Y(0,T)=Y(L,T)=0 \tag{3a}$$

$$Y'(0,T)=Y'(L,T)=0 \tag{3b}$$

$$U(0,T)=U(L,T)=0 \tag{3c}$$

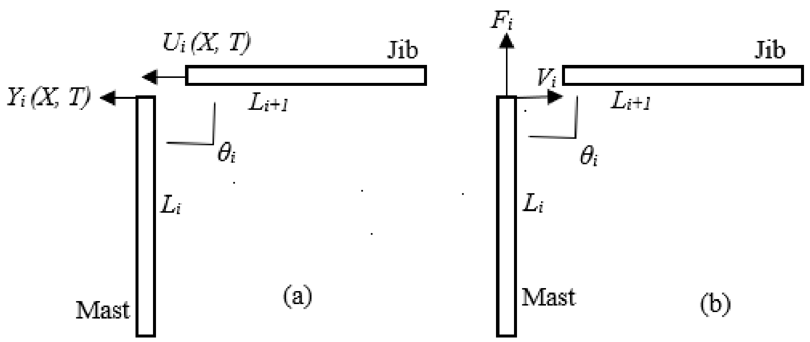


Figure 3. (a) Displacement compatibility, (b) Force compatibility

The transverse as well as the longitudinal motions at the end of the mast before the jib angle constrain the motions of the jib. Hence the ''compatibility conditions'' ensures continuities in the transverse and longitudinal displacement, slope, bending moment, shear force and axial force, respectively from the mast to the jib at angle θ_i.. The compatibility requirements in terms of displacement across the mast from the jib angle θ_i.: Y_i and U_i are transverse and longitudinal displacements of jib at point B as shown in Figure 3(a) and which can be expressed as the force compatibility requirements across the jib angle θ_i.: V_i and F_i are shear and axial forces of the jib at point B as shown in Figure 3(b). The mast is the i-th while the jib is the $i+1$.

$$Y_{i+1}(X_i^+,T) = -Y_i(X_i^-,T)\cos\theta_i + U_i(X_i^-,T)\sin\theta_i \quad \text{Displacement continuity} \tag{4a}$$
$$U_{i+1}(X_i^+,T) = -Y_i(X_i^-,T)\sin\theta_i + U_i(X_i^-,T)\cos\theta_i \quad \text{Displacement continuity} \tag{4b}$$
$$Y'_{i+1}(X_i^+,T) = Y'_i(X_i^-,T) \quad \text{Slope continuity} \tag{4c}$$
$$Y''_{i+1}(X_i^+,T) = Y''_i(X_i^-,T) \quad \text{Moment continuity} \tag{4d}$$
$$EIY'''_{i+1}(X_i^+,T) = -EIY'''_i(X_i^-,T)\cos\theta_i - EIU_i(X_i^-,T)\sin\theta_i \quad \text{Shear continuity} \tag{4e}$$
$$EIU'_{i+1}(X_i^+,T) = EIY'''_i(X_i^-,T)\sin\theta_i - EIU'_i(X_i^-,T)\cos\theta_i \quad \text{Axial force continuity} \tag{4f}$$

where the symbols X_i^+ and X_i^- refers to the mast and jib above and below the angle between the two at point B referred to as X_i respectively. The assumptions in the mentioned compatibility conditions are the same as the normal analysis of the transverse vibrations of Euler–Bernoulli beam and the axial vibrations of a rod. The angle between the mast and jib is also assumed to be constant during the motions of the system.

From the above, the following quantities are introduced:

$$y = \frac{Y}{L},\ x = \frac{X}{L},\ u = \frac{U}{L},\ t = \frac{T}{L},\ l_i = \frac{L_i}{L},\ x_i = \frac{X_i}{L} \tag{5}$$

Thus, in each beam, Equations (1) and (2) can be expressed in a non-dimensional form as

$$\frac{EI}{L^3}\frac{\partial^4 y_i(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_i(x,t)}{\partial t^2} = 0, X_{i-1} < X < X_i, i = 1,2\ (k+1). \tag{6}$$
$$\frac{E}{L}\frac{\partial^2 u_i(x,t)}{\partial x^2} + \rho\frac{\partial^2 u_i(x,t)}{\partial t^2} = 0, X_{i-1} < X < X_i, i = 1,2\ (k+1). \tag{7}$$

The non-dimensional ''compatibility conditions'' from the mast to the jib angle are (from Equations (4a) – (4f))

$$y_{i+1}(x_i^+,t) = -y_i(x_i^-,t)\cos\theta_i + u_i(x_i^-,t)\sin\theta_i \tag{8a}$$
$$u_{i+1}(x_i^+,t) = -y_i(x_i^-,t)\sin\theta_i + u_i(x_i^-,t)\cos\theta_i \tag{8b}$$
$$y'_{i+1}(x_i^+,T) = y'_i(x_i^-,t) \tag{8c}$$
$$y''_{i+1}(x_i^+,t) = y''_i(x_i^-,t) \tag{8d}$$
$$y'''_{i+1}(x_i^+,t) = -y'''_i(x_i^-,t)\cos\theta_i - \frac{AL^2}{I}u_i(x_i^-,t)\sin\theta_i \tag{8e}$$
$$u'_{i+1}(x_i^+,y) = \frac{I}{AL^2}y'''_i(x_i^-,t)\sin\theta_i - u'_i(x_i^-,t)\cos\theta_i \tag{8f}$$

where $i = 1,2$. With the 1 standing for the mast and 2 for the jib. Similarly, the non-dimensional boundary conditions from Equations (3a) – (3c), for point B having a fixed–fixed ends, is written as

$$y(0,t) = 0, \quad y(l,t) = 0 \tag{9a}$$
$$y'(0,t) = 0, \quad y'(l,t) = 0 \tag{9b}$$

$$u(0,t)=0, \quad u(l,t)=0 \tag{9c}$$

2.2 Calculation of the Eigen Solutions

The solutions of the other boundary conditions can then be obtained through a same procedure. Using the separable solutions: $y_i(x,t)=w_i(x)e^{i\omega t}$ and $u_i(x,t)=\upsilon_i(x)e^{i\omega t}$ in Equations (6) and (7) will lead to the associated eigenvalue problem,

$$w_i''''(x)-\lambda^4 w_i(x)=0, x_{i-1}<x<x_i, i=1,2\ (k+1). \tag{10}$$
$$\upsilon_i''(x)-\gamma^2 \upsilon_i(x)=0, x_{i-1}<x<x_i, i=1,2\ (k+1). \tag{11}$$

where

$$\lambda^4=\frac{\rho A L^4\omega^2}{EI} \text{ and } \gamma^2=\frac{\rho L\omega^2}{E} \tag{12}$$

From Equation (12), the relationship between λ and γ is expressed as

$$\gamma=\lambda^2\frac{1}{L}\sqrt{\frac{I}{A}}=a\lambda^2 \tag{13}$$

where $a=(1/L)\sqrt{I/A}$ is a constant. From Equations (8a) – (8f), the corresponding compatibility conditions from the mast to the jib angle gives

$$w_{i+1}(x_i^+)=-w_i(x_i^-)\cos\theta_i+u_i(x_i^-)\sin\theta_i \tag{14a}$$
$$\upsilon_{i+1}(x_i^+)=-\upsilon_i(x_i^-)\sin\theta_i+\upsilon_i(x_i^-)\cos\theta_i \tag{14b}$$
$$w'_{i+1}(x_i^+,T)=w'_i(x_i^-,t) \tag{14c}$$
$$w''_{i+1}(x_i^+)=w''_i(x_i^-) \tag{14d}$$
$$w'''_{i+1}(x_i^+)=-w'''_i(x_i^-)\cos\theta_i-\frac{AL^2}{I}\upsilon_i(x_i^-)\sin\theta_i \tag{14e}$$
$$\upsilon'_{i+1}(x_i^+)=\frac{I}{AL^2}w'''_i(x_i^-)\sin\theta_i-\upsilon'_i(x_i^-)\cos\theta_i \tag{14f}$$

For $i=1,2..$with the 1 standing for the mast and 2 for the jib, the boundary conditions, from Equations (9a) – (9c), are

$$w(0)=0, \quad w(l)=0 \tag{15a,b}$$
$$w'(0)=0, \quad w'(l)=0 \tag{15c,d}$$
$$\upsilon(0)=0, \quad \upsilon(l)=0 \tag{15e,f}$$

A closed-form solution of this eigenvalue problem is obtained by employing transfer matrix methods as established in [9]. The solutions of Equations (10) and (11), for the mast and jib, are

$$w_i(x)=A_i\cos\lambda(x-x_{i-1})+B_i\sin\lambda(x-x_{i-1})+C_i\cosh\lambda(x-x_{i-1})+D_i\sinh\lambda(x-x_{i-1}) \tag{16}$$
$$\upsilon_i(x)=E_i\cos\gamma(x-x_{i-1})+F_i\sin\gamma(x-x_{i-1}) = E_i\cosh a\lambda^2(x-x_{i-1})+F_i\sinh a\lambda^2(x-x_{i-1}) \quad x_{i-1}<x<x_i, i=1,2. \tag{17}$$

where A_i, B_i, C_i, D_i, E_i and F_i are constants associated with the ith segment ($i=1,2\ (k+1)$). The constants in the $(i+1)$th segment A_{i+1} B_{i+1} C_{i+1} D_{i+1} E_{i+1} and F_{i+1} are related to those in the ith segment A_i, B_i, C_i, D_i, E_i and F_i through the compatibility conditions in Equations (14a) – (14f), which can be expressed as

$$\begin{Bmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \\ E_{i+1} \\ F_{i+1} \end{Bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} \\ \vdots & & & & & \\ \vdots & & & & & \\ t_{61} & t_{62} & t_{63} & t_{64} & t_{65} & t_{66} \end{bmatrix}^i \begin{Bmatrix} A_i \\ B_i \\ C_i \\ D_i \\ E_i \\ F_i \end{Bmatrix} = T^i{}_{6\times 6} \begin{Bmatrix} A_i \\ B_i \\ C_i \\ D_i \\ E_i \\ F_i \end{Bmatrix} \tag{18}$$

where $T^i{}_{6\times 6}$ is the 6×6 transfer matrix which depends on the eigenvalue λ; for which the elements are given in [10]. Through repeated applications of Equation (18), the six constants in mast A_1, B_1, C_1, D_1, E_1

and F_1 can be mapped into those of the jib, hence the number of independent constants of the entire system are reduced to six as:

$$\{A_2 \quad B_2 \quad C_2 \quad D_2 \quad E_2 \quad F_2\}^T = T_{6\times6}\{A_1 \quad B_1 \quad C_1 \quad D_1 \quad E_1 \quad F_1\}^T \tag{19}$$

These six remaining constants A_1, B_1, C_1, D_1, E_1 and F_1 can be determined through the satisfaction of the boundary conditions in Equation (15a) - (15f). For the system, Equations (16), (17), (15a), (15c) and (15e) lead to

$$B_1 + D_1 = 0\,,\ A_1 + C_1 = 0\,,\ F_1 = 0 \tag{20a-c}$$

For satisfaction of the boundary conditions of Equations (16) and (17) at the mast support, Equations (15b), (15d) and (15f), requires

$$-A_2 \sin\lambda l_2 + B_2 \cos\lambda l_2 + C_2 \sinh\lambda l_2 + D_2 \cosh\lambda l_2 = 0 \tag{20d}$$

$$A_2 \cos\lambda l_2 + B_2 \sin\lambda l_2 + C_2 \cosh\lambda l_2 + D_2 \sinh\lambda l_2 = 0 \tag{20e}$$

$$E_2 \sinh a\lambda^2 l_2 + F_2 \cosh a\lambda^2 l_2 = 0 \tag{20f}$$

This is expressed in a matrix form as

$$B_{3\times6}\{A_2 \quad B_2 \quad C_2 \quad D_2 \quad E_2 \quad F_2\}^T = 0 \tag{21}$$

where

$$B_{3\times6} = \begin{bmatrix} -\sin\lambda l_{k+1} & \cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ \cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sinh a\lambda^2 l_{k+1} & \cosh a\lambda^2 l_{k+1} \end{bmatrix} \tag{22}$$

Substituting Equation (19) into Equation (21) and use of Equations (20a) - (20c) gives

$$B_{3\times6}\, T_{6\times6}\{A_1 \quad B_1 \quad C_1 \quad D_1 \quad E_1 \quad F_1\}^T = 0$$

Further written as

$$B_{3\times6}\, T^i{}_{6\times6}\{A_1 \quad B_1 \quad -A_1 \quad -B_1 \quad E_1 \quad 0\}^T = 0 \tag{23}$$

where

$$R_{3\times6} = B_{3\times6} T_{6\times6}{}^{k} T_{6\times6}{}^{k-1} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ \vdots \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \end{bmatrix}$$

Hence, this gives a non-trivial solution which requires

$$\det\begin{bmatrix} r_{11}(\lambda) - r_{13}(\lambda) & r_{12}(\lambda) - r_{14}(\lambda) & r_{15}(\lambda) \\ r_{21}(\lambda) - r_{23}(\lambda) & r_{22}(\lambda) - r_{24}(\lambda) & r_{25}(\lambda) \\ r_{31}(\lambda) - r_{33}(\lambda) & r_{32}(\lambda) - r_{34}(\lambda) & r_{35}(\lambda) \end{bmatrix} = 0 \tag{24}$$

The determinant gives the characteristic equation for the solution of the eigenvalues λ_n. The solution of this equation with Newton–Raphson iterations, using the method shown in [10] gives the eigenvalues. This coefficients of the eigen functions of the equation, $w_n(x)$ and $\upsilon_n(x)$ can then be obtained using a back-substitution into Equations (23) and (18), and then Equations (16) and (17). Applying Equations (15a – f) into Equation (20d - f) for the jib crane leads to

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -\sin\lambda l_{k+1} & -\cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ -\cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\cos a\lambda^2 l_{k+1} & \sin a\lambda^2 l_{k+1} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \\ E_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \\ E_2 \\ F_2 \end{bmatrix} \tag{25}$$

For a non-trivial solution of Equation (25), determinant of coefficients of constants A_1, B_1, C_1, D_1, E_1 and E_1 must be zero. That is

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -\sin\lambda l_{k+1} & -\cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ -\cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\cos a\lambda^2 l_{k+1} & \sin a\lambda^2 l_{k+1} \end{vmatrix} = 0 \tag{26}$$

Expanding the determinant gives the frequency equation

$$-2\cos a\lambda^2 l_{k+1}(\cos\lambda l_{k+1}\cosh\lambda l_{k+1}+1)=0 \tag{27}$$

This is similar to equation of a cantilever beam in [9]. Thus, $\lambda l_{k+1}=\lambda l$ for this case and same solution applies as:

$$\cos\lambda l\cosh\lambda l+1=0 \tag{28}$$

The possible solution of Equation (28) is $\cos\lambda l=0$, which gives

$$\lambda_n=\frac{(2n-1)\pi}{2} \tag{29}$$

From Equations (29) and (13), the natural frequency is:

$$\omega_n=\left(\frac{(2n-1)\pi}{2}\right)^2\frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \tag{30}$$

which can be written as $\omega_n=\beta_i l\sqrt{\frac{EI}{\rho A}}$ where $\beta_i l=\left(\frac{(2n-1)\pi}{2}\right)^2\frac{1}{l^2}$

There are infinite number of natural frequencies associated with a continuous structure as it possesses infinite degrees of freedom. The associate mode shapes are obtained by

$$W_n(x)=\sinh\lambda_n x-\sin\lambda_n x-\alpha(\cosh\lambda_n x-\cosh\lambda_n x)$$
$$\text{where } \alpha=\left[\frac{\sinh\lambda_n l+\sin\lambda_n l}{\cosh\lambda_n l+\cosh\lambda_n l}\right] \qquad n=1,\ 2,\ 3,\ldots \tag{31}$$

The response of the beam is given as a linear combination of mode shapes.

2.3 Forced Vibration of the Jib

The principle of superposition is used to determine the solution of the forced vibration of the jib as a cantilever beam as shown in Figure 4a. For this, the deflection of the beam is assumed as

$$w(x,t)=\sum_{n=1}^{\infty}W_n(x)q_n(t) \tag{32}$$

where $q_n(t)$ is the generalized coordinate in the n-th mode and $W_n(x)$ is the n-th normal mode or characteristic function satisfying the differential equation [10].

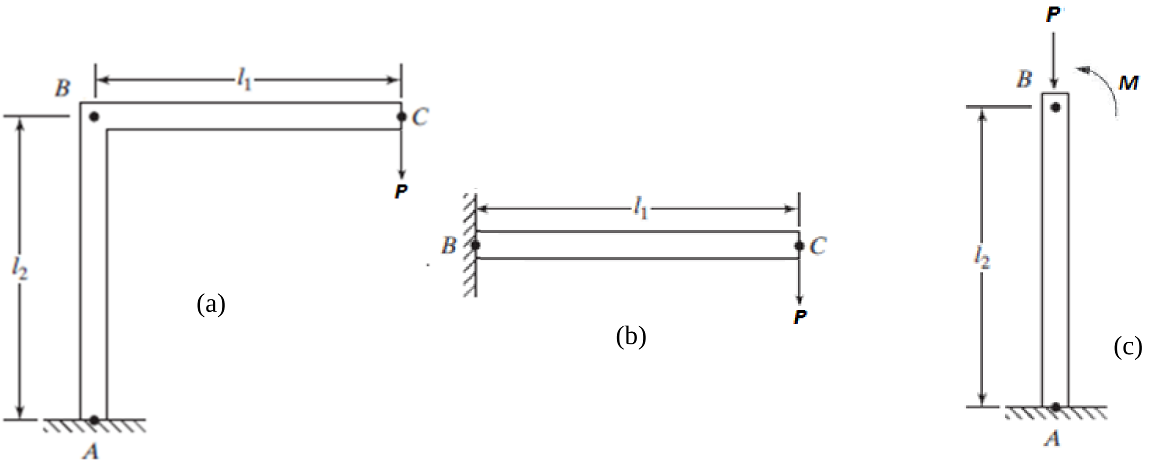


Figure 4. (a) Schematic of the jib crane, (b) The jib under loading, (c) The mast under loading

$$EI\frac{d^4W_n}{dx^4}(x)-\omega^2\rho AW_n(x)=0; n=1,2. \tag{33}$$

Substituting Equation (32) in Equation (33) gives

$$\sum_{n=1}^{\infty}EI\frac{d^4\mathrm{W}_n(x)}{dx^4}+\sum_{n=1}^{\infty}\rho A(x)\,\mathrm{W}_n(x)\frac{d^2q_n(t)}{dt^2}=f\left(x,t\right) \tag{34}$$

From Equation (33), Equation (34) can be written as

$$\sum_{n=1}^{\infty}\omega^2W_n(x)q_n(t)+\sum_{n=1}^{\infty}\mathrm{W}_n(x)\frac{d^2q_n(t)}{dt^2}=\frac{1}{\rho A}f\left(x,t\right) \tag{35}$$

Multiplying Equation (35) throughout by W_m, integrating from 0 to l and using orthogonality condition gives

$$\frac{d^2q_n(t)}{dt^2}+\omega^2q_n(\mathrm{t})=\frac{1}{\rho Ab}Q_n(\mathrm{t}) \tag{36}$$

where $Q_n(t)$ is the generalized forced corresponding to $q_n(t)$

$$Q_n(\mathrm{t})=\int_0^l f(x,\mathrm{t})\,\mathrm{W}_n(x)dx \tag{37}$$

and the constant is given by

$$b=\int_0^l f(x,\mathrm{t})\,\mathrm{W}_n(x)dx \tag{38}$$

Equation (32) is similar to the equation of motion of undamped single degree of freedom system. Using Duhamel integral, the solution of the Equation (32) can be expressed as

$$q_n(t)=A_n\cos\omega_nt+B_n\sin\omega_nt+\frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(\mathrm{t}-\tau)d\tau \tag{39}$$

2.4 Response of the Jib Subjected to Harmonic Force

From Equation (36), the generalized coordinate $q_n(t)$ is given by

$$\frac{d^2q_n(t)}{dt^2}+\omega^2q_n(\mathrm{t})=\frac{1}{\rho Ab}Q_n(\mathrm{t}) \tag{40}$$

where

$$Q_n(\mathrm{t})=\int_0^l f(x,\mathrm{t})\,\mathrm{W}_n(x)dx=\mathrm{F}(\mathrm{t})\,\mathrm{W}_n(l) \tag{41}$$

The steady-state solution of Equation (40) is given by

$$q_n(t)=A_n\cos\omega_nt+B_n\sin\omega_nt+\frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(\mathrm{t}-\tau)d\tau \tag{42}$$

where $b=\int_0^l \mathrm{W}^2{}_n(x)dx=l$

$$\int_0^t Q_n(\tau)\sin\omega_n(\mathrm{t}-\tau)d\tau=PW_n(l)\int_0^t e^{\omega\tau}\sin\omega_n(\mathrm{t}-\tau)d\tau=PW_n(l)e^{\omega\tau}\int_{t-\tau=0}^{t-\tau=t}e^{\omega\tau}\sin\omega_n(\mathrm{t}-\tau)(-d\tau) \tag{43}$$

Using the formula

$$\int e^{ax}\sin bxdx=\frac{1}{a^2+b^2}e^{ax}\left\{a\sin bx-b\sin bx\right\} \tag{44}$$

Equation (43) can be evaluated to obtain

$$q_n(t)=\frac{PW_n(l)}{\rho Al\omega_n(\omega_n^2+\omega^2)}\left\{\omega_ne^{\omega t}+\omega\sin\omega_nt-\omega_n\cos\omega_nt\right\} \tag{45}$$

2.5 Response of the Mast Subjected to a Moment at the Free End

From Equation (37), the generalized force $Q_n(t)$ becomes

$$Q_n(t) = M_0 \left.\frac{dW_n}{dx}\right|_{x=l} \tag{46}$$

where

$$\left.\frac{dW_n}{dx}\right|_{x=l} = \beta_n(\cos\beta_n l - \cosh\beta_n l) + \alpha_n\beta_n(\sin\beta_n l + \sinh\beta_n l) \tag{47}$$

The steady-state response of the beam under the action of the moment is given by Equation (32) with

$$q_n(t) = \frac{1}{\rho A b\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(\mathrm{t}-\tau)d\tau$$

$$q_n(t) = \frac{1}{\rho A b\omega_n} M_0 \left.\frac{dW_n}{dx}\right|_{x=l} \int_0^t \sin\omega_n(\mathrm{t}-\tau)d\tau \tag{48}$$

where $b = \int_0^l \mathrm{W}^2{}_n(x)dx = l$

Noting that $\int_0^t \sin\omega_n(\mathrm{t}-\tau)d\tau = \frac{1}{\omega_n}(1-\cos\omega_n t)$

The steady-state response can be written as

$$q_n(t) = \frac{1}{\rho A b\omega_n} M_0 \left.\frac{dW_n}{dx}\right|_{x=l} (1-\cos\omega_n t) \tag{49}$$

Thus, the combined response of the jib crane due to the harmonic force and moment is

$$q_n(t) = \frac{pW_n(l)}{\rho A l\omega_n(\omega_n^2+\omega^2)}\left\{\omega_n e^{\omega t} + \omega\sin\omega_n t - \omega_n\cos\omega_n t\right\} + \frac{1}{\rho A b\omega_n} M_0 \left.\frac{dW_n}{dx}\right|_{x=l} (1-\cos\omega_n t) \tag{50}$$

3. RESULTS AND DISCUSSION

3.1 Numerical parameters

This work used a standard numerical data obtained from [9] for a 5-ton model. I-cross section beam, $L = 6.096$ m, $A = 0.603$ m, $B = 0.179$ m, $C = 0.015$ m and $D = 0.011$ m with structural steel (A36) having $\rho = 7800$ kg/m^3, $E = 2 \times 10^{11}$ GPa and $\vartheta = 0.3$. The force acting is $P = 4905$ N and a motor speed of 7.4 m/min on 0.326 m drum.

3.2 The Natural Frequencies of the Jib Crane

The system parameters were calculated as $\omega = 0.3783$ rad/s, $A = 0.01167$ m^2, $M = 555.4$ kg, $I = 6.3672 \times 10^{-4}$ m4, $k = 1683.1$ kN/m, $\omega_n = 55.049$ rad/s, $\omega/\omega_n = 0.0069$, and maximum displacement as $X_p = 29.144$ mm. The first three natural frequencies are obtained by solving Equation (30) and tabulated as in Table 1.

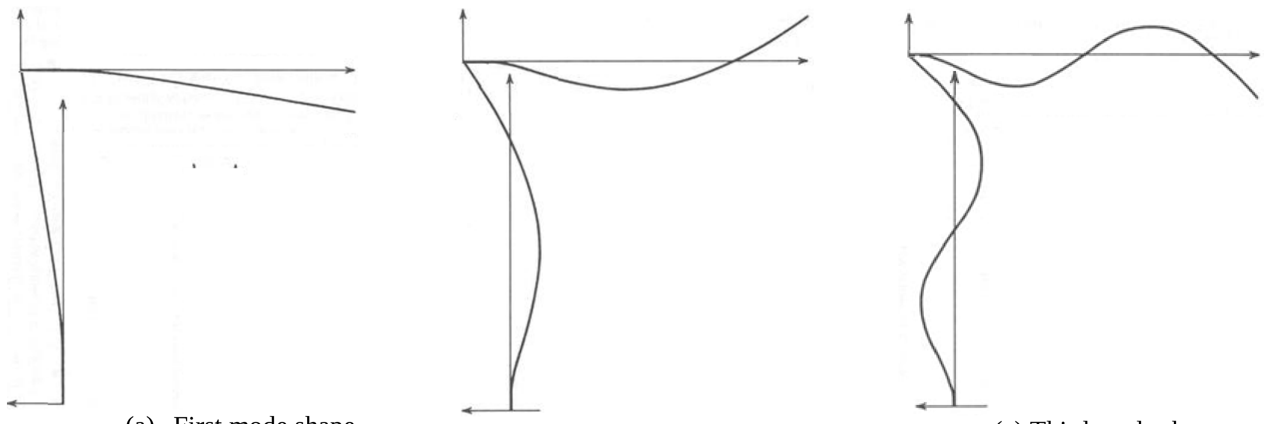
Table 1 shows the first three natural frequencies of the jib crane. This result is almost similar to the solution of a cantilever beam with same boundary conditions as derived in Equation (30). The result tallies with [8] as a good indication of the effective of this approaching of treating the jib crane as a frame structure.

3.3 Mode shapes of the Jib Crane

The mode shapes of the jib crane are shown in Figure 5(a)-(c) from Equation (31). It shows that each of the mast and the jib behaves as an independent cantilever beam. The mast is fixed at the base, but the other end is free in its axis while forming a fixed support for the jib.

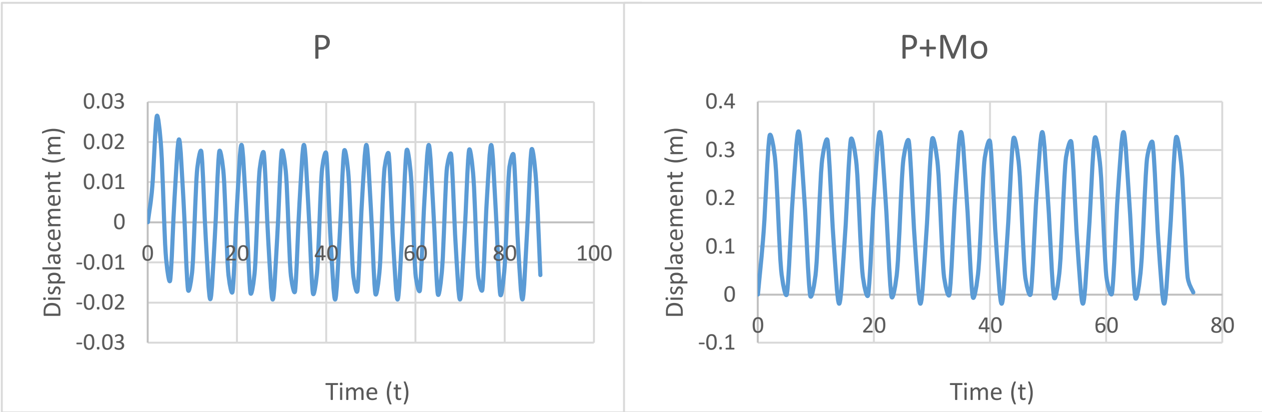
Table 1. Modal analysis frequencies

Mode	$\beta_i l$	Theoretical (Hz)
1	1.8751	17.8086
2	4.6941	111.4336
3	7.8548	312.4961



(a) First mode shape (b) Second mode shape (c) Third mode shape

Figure 5. Mode shapes



(a) Jib (b) Mast

Figure 6. Time response

3.4 The Response of the Jib Crane to the Force

The response of the jib to the force at the end of the jib which is the load carried by the jib crane from Equation (45) is shown in Figure 6(a) with a maximum displacement of about 26 mm and a mean steady displacement of less than 20 mm. The response of the mast due to the combined effect of the harmonic force and the moment acting on it from Equation (50) is shown in Figure 6(b) with a mean displacement of about 32 mm. The response of the mast is higher due to the combined displacement of the jib and mast from the superposition effect.

4. CONCLUSION

This paper explores an effective method of analysing the vibration of a jib crane by utilizing vibration analysis of frame structures with the aim of finding the amplitude, frequency and mode shapes of a jib crane. The approach shows that the jib crane can be modelled and treated as a frame structure in analysing its vibration parameters. Frequency, mode shape and amplitude of the cantilever beam were analysed as functions of vibration parameters. An approximate assumed mode equation was derived using expansion of the modes shape as the expansion function for the beams. These parameters are functions of the boundary conditions of the relationship between the mast and the jib in the system. The results from this approach shows a feasible and practical approach to the utilization of vibration of frame structures in analysing the vibration parameters of a jib crane. Natural frequencies, mode shapes, amplitude and response of vibration of the jib crane conformed well to already established works.

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