

# Finite Element Modelling of Flux Density Distribution in Three Phase Transformer Under Nonlinear Load Condition

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**Abstract:** For a transformer under linear load operation, the current into the magnetic material core consists of only fundamental component. However under nonlinear load, the current consists of fundamental and harmonic components with certain percentage of harmonic distortion. It changes flux density as the current is no longer sinusoidal. This paper explores the in-similarity of flux density reading of three phase three limb power transformer under linear and nonlinear load conditions. A finite element model is designed and solved using Opera, a Finite Element solver in order to illustrate flux density values under variation of loads. Results of this paper points to the need to make changes towards dimensions of power transformer core in order to avoid potential malfunction.

**Keywords:** Current harmonic; Flux density; Nonlinear loads; Total Harmonic Distortion (THD); Transformer Core.

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## 1. INTRODUCTION

The power electronic systems connected to mains generated distorted current non-sinusoidal and contain high percentage of harmonics. Some examples of power electronic equipments being used in power system nowadays are adjustable speed drive (ASD), variable frequency drive (VFD), personal computer, server, air conditioner, switch mode power supply (SMPS), etc. These nonlinear loads inject current of high amplitude short pulses and contribute to significant distortion to the initial sinusoidal current waveform resulting in existence of current harmonics [1].

The issues with current harmonics at power system are well documented [1-4] and regularly results in cases such as blown fuses and malfunction on circuit breaker (CB) [5]. In a three phase power system, overheating of neutral conductor is caused by large amount of current in it. Standard power system equipments such as power conductor and power transformer are often directly impacted to current harmonics. Generally, they are designed and manufactured under rated current but with high distortion in current which mean high rms reading of current will resulting in overload. This will lead to significant higher losses and possible risk of malfunction [6].

This paper aims to investigate the effect of various characteristic of loads to the operation of power transformer. Specifically, it will explore the effect of flux density distribution as the components of current changes from linear to nonlinear load. Cobham plc Vector Field Opera is the Finite Element (FE) solver used for the purpose of this paper. A real time 2 kVA three phase power transformer is remodeled and solved in Finite Element Analysis (FEA). Analysis includes formation of equation that relates flux density and the state of current around magnetic materials that constitute the transformer core. Flux density at the core under various current Total Harmonic Distortion ( $THD_i$ ) percentages is illustrated in graphs and tabulations. With the knowledge of flux density values under various current harmonic distortions, irregularities of flux density can be mapped. It has been known that high harmonic current causes additional loss and heating all over transformer, and this relates to the high concentration of flux at one or many particular areas at the unit. In order to reduce high concentration of flux, this paper hopes to produce a new design of transformer.

## 2. THREE PHASE TRANSFORMER MAGNETIC CIRCUITS

Both magnetic and electrical equivalent circuits can be analyzed in tandem in order to determine the similarity between electric and magnetic operating losses. Figure 1 shows an electrical equivalent circuit of a transformer.

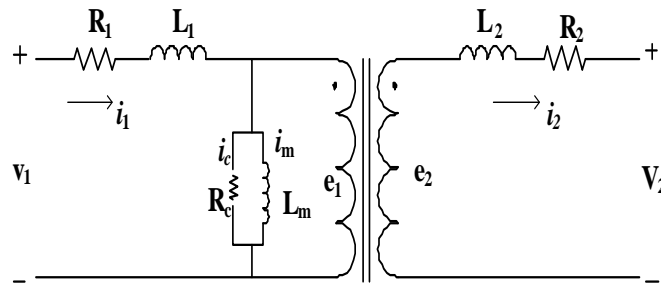


Figure1. Equivalent circuit per phase of a three phase transformer

Where:

- $V_1$  is the input voltage (primary input voltage)
- $R_1$  is the primary winding resistance
- $L_1$  is the primary leakage inductance
- $i_1$  is the primary current
- $e_1$  is the primary induced voltage
- $R_2$  is secondary winding resistance
- $L_2$  is the secondary leakage inductance
- $i_2$  is secondary current
- $e_2$  is secondary induced voltage
- $V_2$  is the output voltage (secondary output voltage)
- $L_m$  is magnetizing inductance
- $i_m$  is magnetizing current
- $i_c$  is core losses current
- $R_c$  is resistance for core losses

Copper losses of transformer  $P_c$  can be expressed as:

$$P_c = (i_1^2 \cdot R_1) + (i_2^2 \cdot R_2) \tag{1}$$

Figure 2 shows the distribution pattern of flux and its magnetic equivalent circuit of three phase power transformer.

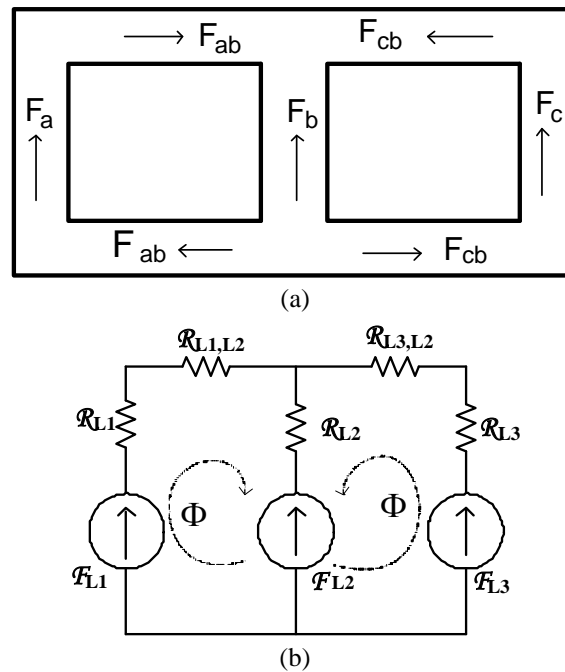


Figure 2. (a) Flux distribution pattern in three phase three limb power transformer (b) Magnetic equivalent circuit

Where:

- $\mathcal{F}$  is the magnetomotive force and is allocated to first, second, and third limb respectively.
- $\mathcal{R}$  is the reluctance and the loop indicates the movement of the flux within the circuit.

Kirchhoff Current Law (KCL) can describe the movement of flux,  $\phi$  with respect to other parameters [7-8,12].

$$\mathcal{F}_{L1} + (-\mathcal{F}_{L2}) = (N \cdot i_{L1} - N \cdot i_{L2}) = \phi_{L1} (\mathcal{R}_{L1}) + \phi_{L1,L2} (\mathcal{R}_{L1,L2}) - \phi_{L2} (\mathcal{R}_{L2}) \quad (2)$$

$$\mathcal{F}_{L3} + (-\mathcal{F}_{L2}) = (N \cdot i_{L3} - N \cdot i_{L2}) = \phi_{L3} (\mathcal{R}_{L3}) + \phi_{L3,L2} (\mathcal{R}_{L3,L2}) - \phi_{L2} (\mathcal{R}_{L2}) \quad (3)$$

By assuming the magnetic flux density is uniformly distributed across core cross sectional area (CSA), flux  $\phi$  is explained as:

$$\phi = B \cdot A \quad (4)$$

of which  $B$  is the core flux density and  $A$  is the core CSA. Flux density,  $B$  is the measure of flux at one particular area,  $A$ . Thus,  $B = \frac{\phi}{A}$ . Now that relation between flux,  $\phi$ , mmf ( $\mathcal{F}$ ), and reluctance,  $\mathcal{R}$  is established by Fitzgerald et al. as in Figure 3 [9]:

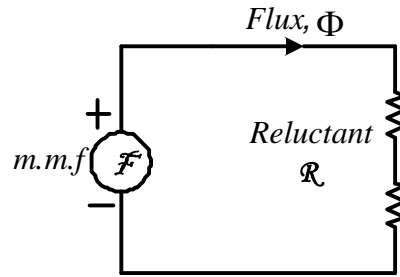


Figure 3. Relation between flux,  $\phi$ , mmf ( $\mathcal{F}$ ), and reluctance,  $\mathcal{R}$

By applying Kirchoff Law, the relation between the three can be established easily. Thus,

$$\mathcal{F} = N \cdot i$$

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_{total}} = \frac{N \cdot i}{\mathcal{R}_{total}} \quad (5)$$

Where  $N$  is the number of turn and  $i$  is the current that flows into the turn. Under linear load, flux density can be described by combining Equations (5) and (4).

$$B = \frac{\phi}{A} = \frac{N \cdot i}{\mathcal{R} \cdot A} \quad (6)$$

Equation (6) proves the dependency of flux density to the state of current. While under operation with linear load, Equation (6) can be used to describe the projection of flux density. However under nonlinear load, Equation (6) no longer applies. This paper looks to IEEE Standard C.57-2008 as the main document in determining rms current under nonlinear load for transformer operation. It insists for the value of rms for non-sinusoidal current to be determined through specific harmonic current distribution. In order to evaluate rms value of harmonic current, this paper referred to the below equation as mentioned in IEEE Std C.57.110-2008 [10-11]. Instantaneous linear load current over time is described as:

$$i(t) = i_1 \sin \omega t \quad (7)$$

Under nonlinear load, distortion in current occurs, thus causing harmonic current to exist with respect to its order. Thus 3<sup>rd</sup>, 5<sup>th</sup>, and 7<sup>th</sup> order of harmonics are represented over time as follows:

$$i_3(t) = i_3 \sin (3\omega t - \delta_3) \quad (8)$$

$$i_5(t) = i_5 \sin (5\omega t - \delta_5) \quad (9)$$

$$i_7(t) = i_7 \sin (7\omega t - \delta_7) \quad (10)$$

Simplification of harmonic components in current as power transformer feeds nonlinear load yields:

$$i_h(t) = i_h \sin (h\omega t - \delta_h) \quad (11)$$

Where:

$h$  is harmonic order

$\omega = 2\pi f$

$\delta$  is displacement angle for particular waveform

Elaboration of harmonic current can be defined as in Equation (12).

$$i(t) = i_1 \sin \omega t + i_3 \sin (3\omega t - \delta_3) + i_5 \sin (5\omega t - \delta_5) + i_7 \sin (7\omega t - \delta_7) + \dots i_n \sin (h\omega t - \delta_h) \quad (12)$$

Total effective load current or rms value of current is the square root of summarization of current component square. It can be represented mathematically as in Equation (13).

$$I_{rms} = \sqrt{\sum_{h=1}^{h=h_{max}} i_h^2} \quad (13)$$

Where  $i_h$  is the current at  $h^{th}$  order harmonic, and this equation is useable until maximum significant harmonic order,  $h_{max}$ . Flux density of transformer under nonlinear load must consider the state of current consisting of harmonic components. Described as in Figure 3, it is very important to notice that flux and its density across an area are dependent towards the value of current,  $i$  as in Equation (14).

$$f(\phi) = f(i) \quad (14)$$

In turn, Equations (11)–(13) stated that rms magnitude of distorted current over time is determined by the sum of all individual harmonic components. Level of distortion which cause the existence of harmonic components in current is then determined in Total Harmonic Distortion in current ( $THD_i$ ), thus the following relation can be established.

$$f(i) = f(THD_i)$$

$$f(\phi) = f(i) = f(THD_i) \quad (15)$$

From Equation (15), it is safe to conclude that flux under linear load is determined by linear load current;  $\phi_1 = i_1$ . However, under nonlinear load where harmonic current exist, the total flux in a transformer is defined as:

$$\phi_T = \phi_1 + \sum \phi_h \quad (16)$$

Where  $\phi_T$  is the total flux,  $\phi_1$  is the fundamental current flux, and  $\phi_h$  is the flux produced by harmonic current.

### 3. FINITE ELEMENT MODELLING AND ANALYSIS

Cobham plc Vector Field Opera is chosen as FE modeling and solver due to its features such as high accuracy of meshing, able to present nonlinear solver, and ability to attach external circuits. The purpose of presenting finite element analysis is to enhance the finding of this paper particularly in elaborating the relation between linear and harmonic current with the state of flux distribution and its density. Several published models have been referred for building a 2D FE model for this paper [7], [13-17]. Figure 4 shows the two-dimensional CSA of three phase transformer with allocation of HV and LV windings.

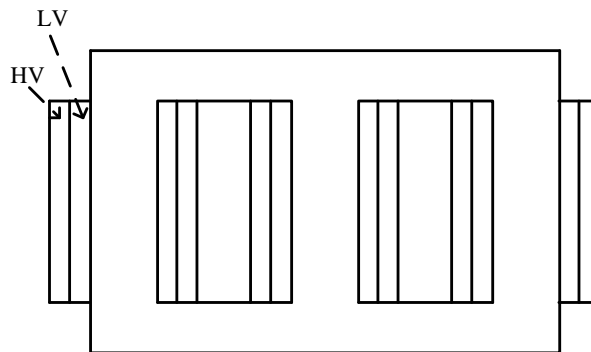


Figure 4. Three phase transformer with HV and LV winding allocation

By applying the above set up, flux distribution around the core can now be observed as illustrated in Figure 5. It shows pattern of flux distribution at power transformer core under rated operation. Crucial regions that contributed to flux suffocations which subsequently caused rise of temperature are the outer limbs as well as the middle limb. They are shown as in Figure 6.

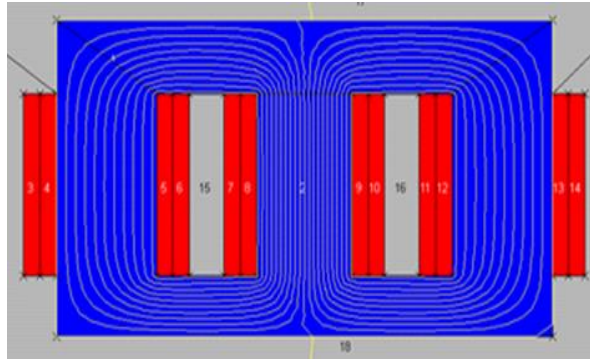


Figure 5. Flux distributions at core

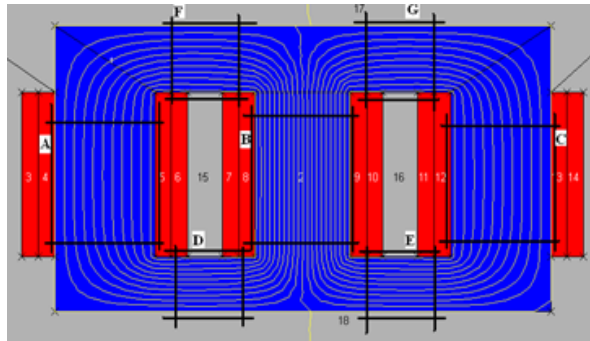


Figure 6. Regions with high flux concentration

Figure 6 shows all regions with high magnetic flux concentration when the windings are supplied with primary and secondary currents. To observe flux distribution at selected regions, simulation of the model must be performed. The contour map POT is chosen for this purpose which means Opera calculates the vector potential which refers to flux distribution around the E-I magnetic core. As for measuring flux values at any spot around the core, solver function of BMOD is utilized. By applying IEEE C.57 2008, rms current with variation of  $THD_i$  (%) can be determined.

#### 4. RESULT AND DISCUSSION

Simulation of a 2 kVA three phase power transformer is done and the analysis of current and flux density at the limbs are completed. The full load current of 2.78 A is inserted under linear load operation, and the content of harmonics components in current is raised. Flux distribution pattern is likely to be similar as in Figure 5 for all kind of harmonic current. This is due to inability of the FE solver to illustrate the additional of lines as harmonic component increases in current. However, under Color Contour Map selection in the solver, the difference in flux density around the magnetic material is visible. The results of flux density at  $THD_i$  equals 0% - 40% and 45% - 50% respectively are shown in Figure 7 and Figure 8. Coloured zone is another option available under Contour Map in order to observe and evaluate flux density across transformer core CSA. Reddish coloured regions indicate high density of flux and the value goes down with respect to the colour becoming bluish.

This paper focuses the middle limb of the transformer in order to differentiate the effect of linear and nonlinear load towards flux density of transformer. Figure 9 and Figure 10 show the flux density reading obtained by FE solver at middle limb for variation of  $THD_i$ .

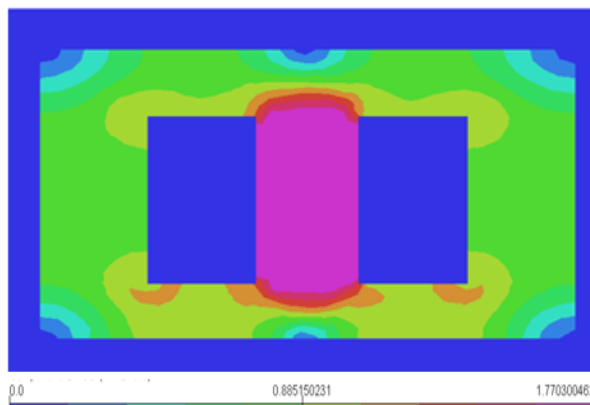


Figure 7. Flux distribution at core CSA for 0% to 40%  $THD_i$

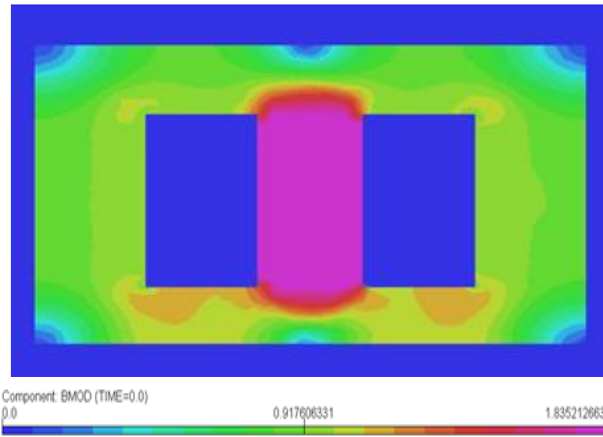


Figure 8. Flux distribution at core CSA for 45% to 50%  $THD_i$

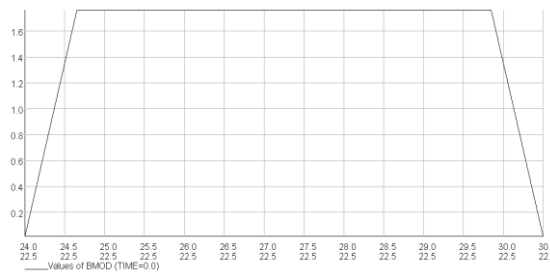


Figure 9. Flux density at transformer middle limb with rated current ( $THD_i = 0\%$ )

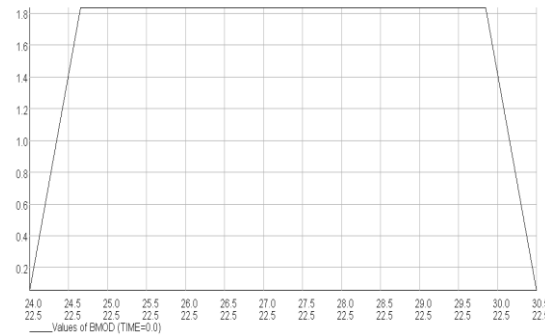


Figure 10. Flux density at transformer middle limb with harmonic current ( $THD_i = 50\%$ )

By using option Fields at A Point function; the exact value of flux density at any given spot across transformer CSA can be obtained. Table 1 shows the tabulation of flux density with respect to  $THD_i$  at full load current, at selected regions from A to G as illustrated in Figure 6.

Table 1. Flux density at selected regions

THD (%)	Flux density (T)						
	A	B	C	D	E	F	G
0	0.86	1.76	0.86	1.03	1.03	0.86	0.86
5	0.87	1.77	0.87	1.04	1.04	0.87	0.87
10	0.87	1.77	0.87	1.04	1.04	0.87	0.88
15	0.88	1.78	0.88	1.05	1.05	0.88	0.88
20	0.88	1.78	0.88	1.05	1.05	0.88	0.88
25	0.88	1.79	0.88	1.06	1.06	0.88	0.88
30	0.89	1.80	0.89	1.06	1.07	0.88	0.89
35	0.89	1.81	0.90	1.07	1.08	0.89	0.89
40	0.90	1.82	0.90	1.08	1.08	0.89	0.90
45	0.91	1.83	0.92	1.1	1.1	0.91	0.91
50	0.91	1.83	0.92	1.1	1.1	0.91	0.91

Table 1 shows the rising pattern of flux density with respect to harmonic content in current. At all measured regions from A to G, the values keep increasing as  $THD_i$  increases and in agreement with Equation (5). Under linear load, the value of current,  $i$  only consist of fundamental component. With number of turn,  $N$  and reluctance of the transformer  $\mathcal{R}_{total}$  is kept constant, flux  $\phi$  is equal to rated value. On the other hand, under nonlinear load, value of current,  $i$  is equal to  $i_1 + \sum i_h$  thus explaining the reason of rising value of flux as tabulated in Table 1.

## CONCLUSION

This paper successfully determines the effect of linear and nonlinear load towards flux density distribution in three phase transformer magnetic material. It clearly shows that under operation with harmonic currents, the flux density distribution will increase at all areas. From 0% to 50%  $THD_i$ , the value of flux density steadily increases up to 5% posing risk of malfunction to the operation of the three phase transformer. At crucial sections such as indicated by Figures 7 and 8, the flux density is highly concentrated which causes heat to build up. At prolonged operation with such condition is detrimental to the health of transformer core and the windings circulates at the limbs. Transformer core can be heated up well above its designed value, and failed the insulations of the core and the circulating windings.

## REFERENCES

- [1] V. A. Ramos Jr, Treating harmonics in electrical distribution systems, *Computer Power and Consulting Corporation*, 1999.
- [2] P. J. A. Ling and C. J. Eldridge, Designing modern electrical systems with transformers that inherently reduce harmonic distortion in a PC-rich environment, *Technical Paper*. Powersmiths International Corporation.
- [3] M. S. Hwang, W. M. Grady and H. W. Sanders, Distribution transformer winding losses due to nonsinusoidal currents, *IEEE Transactions on Power Delivery*, 2(1), 140–146, 1987.
- [4] M. Mehrdad, E. K. Stanek and A. S. Jannati, Influence of voltage and current harmonics on behavior of electric devices, *International Journal of Advanced Science and Technology*, 75(7), 21–32, 2015.
- [5] S. Edwards, D. Bobick and S. Weinzierl, Impact of harmonic current on energy meter calibration, *IEEE 2011 EnergyTech*, Cleveland, USA, 2011, pp. 1–6.
- [6] M. S. Taci and M. H. Sarul, Investigation of a sinusoidal supplied transformer under linear and nonlinear loading condition, *TENCON 2008*, Hyderabad, India, 2008, pp. 1–6.
- [7] S. Masri, A. Mokhzaini and M. Kamarol, Flux distribution at three phase power transformer core under nonlinear load operation, *International Review on Modelling and Simulation*, 5(2), Part B, 872–878, 2012.
- [8] S. G. Abdulsalam, W. Xu, W.L.A. Neves and X. Liu, Estimation of transformer saturation characteristics from inrush current waveforms, *IEEE Transactions on Power Delivery*, 21(1), 170–177, 2006.
- [9] A. E. Fitzgerald, C. Kingsley Jr and S. D. Umans, *Electric Machinery International Edition*, McGraw Hill.
- [10] IEEE recommended practice for establishing transformer capability when supplying nonsinusoidal load currents, ANSI/IEEE C.57.110-Revision 2008.
- [11] T. Hoevenaars, K. LeDoux and M. Colosino, Interpreting IEEE STD 519 and meeting its harmonic limits in VFD applications, *IEEE Industry Applications Society 50th Annual Petroleum and Chemical Industry Conference*, Houston, USA, 2003, pp. 145–150.
- [12] J. Faiz, B. M. Ebrahimi and M. Ghofrani, Mixed derating of distribution transformers under unbalanced supply voltage and nonlinear load conditions using TSFEM, *IEEE Transactions on Power Delivery*, 25(2), 780–789, 2010.
- [13] C. Hernandez and M. A. Arjona, Design of distribution transformer based on a knowledge-based system and 2D finite elements, *Finite Elements in Analysis and Design*, 43(8), 659–665, 2007.
- [14] J. A. Rojas and J. R. Varela, Numeric simulation of the magnetic flux in a power transformer, *IEEE Transmission and Distribution Conference and Exposition*, Caracas, Venezuela, 2006, pp. 1–6.
- [15] G. E. Mechler and R. S. Girgis, Magnetic flux distributions in transformer core joints, *IEEE Transactions on Power Delivery*, 15(1), 198–203, 2000.
- [16] T. Noda, H. Nakamoto and S. Yokoyama, Accurate modelling of core-type distribution transformers for electromagnetic transient studies, *IEEE Transactions on Power Delivery*, 17(4), 969–976, 2002.
- [17] M. B. B. Sharifian, J. Faiz, S. A. Fakheri, and A. Zraatparvar, Derating of distribution transformers for non sinusoidal load currents using finite element method, *10<sup>th</sup> IEEE International Conference on Electronics, Circuits and Systems*, Sharjah, United Arab Emirates, 2003, pp. 754–757.